Name________________________ ID__________________

1-10	/60	12	/15
11	/20	13	/15
Total			/110

MATH 251 Final Exam Spring 2007

Sections 509 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. Consider the triangle with vertices $A = (2,4)$, $B = (1,1)$ and $C = (0,3)$.
Find the angle at B.

a. $30°$

b. $45°$

c. $60°$

d. $120°$

e. $135°$

2. For the "twisted cubic" **curve** $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ find the tangential acceleration $a_T = \hat{T} \cdot \vec{a}$.

a. $4t + 8t^2$

b. $\dfrac{1}{4t + 8t^2}$

c. $4t$

d. $\dfrac{4t}{1 + 2t^2}$

e. $\dfrac{1}{1 + 2t^2}$

3. Find the linear approximation to $f(x,y) = (x^2 + 4)(y^3 + 1)$ at $(x,y) = (2,1)$.
Use it to estimate $f(2.1, 1.1)$.

a. 19.6

b. 19.2

c. 16.2

d. 12.8

e. 5.87

4. Find the equation of the plane tangent to the surface $3x\sin z + y\cos z = 4$ at $(x,y,z) = \left(\sqrt{2}, \sqrt{2}, \frac{\pi}{4}\right)$.
What is the z-intercept?

a. $\frac{\pi}{2}$

b. $8 + \pi$

c. $4 + \frac{\pi}{2}$

d. $2 + \frac{\pi}{4}$

e. $1 + \frac{\pi}{8}$

5. Find the arc length of the curve $\vec{r}(t) = (2t^2, t^3)$ between $t = 0$ and $t = 1$.

a. $\frac{31}{27}$

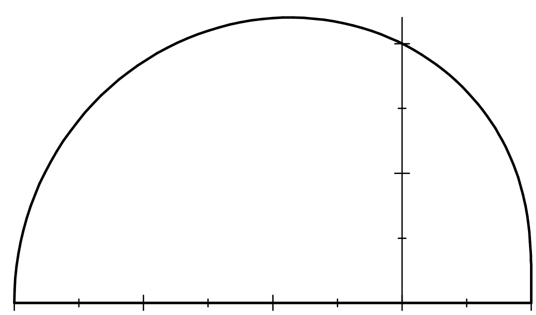
b. $\frac{61}{27}$

c. $\frac{91}{27}$

d. $\frac{31}{9}$

e. $\frac{61}{9}$

6. Find the mass of the region inside the upper half of the limaçon $r = 2 - \cos\theta$ if the surface density is $\rho = y$.



a. $\frac{5}{3}$

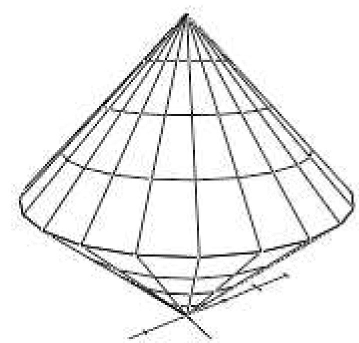
b. $\frac{10}{3}$

c. $\frac{13}{3}$

d. $\frac{15}{3}$

e. $\frac{20}{3}$

7. Find the mass of the solid between the cones $z = r$ and $z = 6 - r$ if the volume mass density is $\rho = z$.



a. 54π

b. 108π

c. 216π

d. $\frac{297}{8}\pi$

e. $\frac{297}{4}\pi$

8. Compute $\int_{\vec{r}} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2x + y + z, 2y + x + z, 2z + x + y)$ along the curve $\vec{r}(t) = (t\cos t, t\sin t, te^{t/\pi})$ between $t = 0$ and $t = \pi$.

HINT: Find a scalar potential and use the Fundamental Theorem of Calculus for Curves.

a. $\pi^2(1 + e^2 - e)$

b. $\pi^2(1 + e^2 - 2e)$

c. $\pi^2(1 + e^2 + e)$

d. $\pi^2(1 + e^2 + 2e)$

e. $\pi^2(1 + e^2)$

9. Compute $\oint_C \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-x^2y + x^3 - y^3, xy^2 + x^3 - y^3)$ counterclockwise around the circle $x^2 + y^2 = 9$.

HINT: Use Green's Theorem.

a. 36π

b. 72π

c. 144π

d. 162π

e. 324π

10. Compute $\iint_{\partial C} \vec{F} \cdot d\vec{S}$ over the complete boundary of the cylinder $x^2 + y^2 \le 4$ for $0 \le z \le 5$

for the vector field $\vec{F} = (x^3 + xy^2, x^2y + y^3, x^2z + y^2z)$.

HINT: Use Gauss' Theorem.

a. $\frac{80}{3}\pi$

b. 40π

c. 100π

d. $\frac{400}{3}\pi$

e. 200π

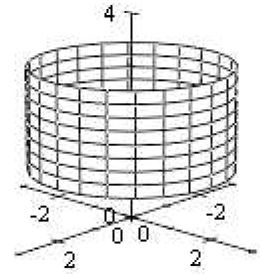
Work Out: (Points indicated. Part credit possible. Show all work.)

11. $\left(20 \text{ points}\right)$ Verify Stokes' Theorem $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint\limits_{\partial C} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = (yz^2, -xz^2, z^3)$ and the cylinder $x^2 + y^2 = 9$ for $1 \le z \le 2$ oriented out.

Be sure to check and explain the orientations.

Use the following steps:



a. The cylindrical surface may be parametrized by $\vec{R}(\theta, z) = (3\cos\theta, 3\sin\theta, z)$.
Compute the surface integral:
Successively find: $\vec{e}_\theta$, $\vec{e}_z$, $\vec{N}$, check orientation, $\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\left(\vec{R}(\theta, z)\right)$, $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

b. Let U be the upper circle. Parametrize U and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, check orientation, $\vec{F}(\vec{r}(\theta))$, $\oint_U \vec{F} \cdot d\vec{s}$.

c. Let L be the lower circle. Parametrize L and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, check orientation, $\vec{F}(\vec{r}(\theta))$, $\oint_L \vec{F} \cdot d\vec{s}$.

d. Combine $\oint_U \vec{F} \cdot d\vec{s}$ and $\oint_L \vec{F} \cdot d\vec{s}$ to get $\oint_{\partial C} \vec{F} \cdot d\vec{s}$.

12. (15 points) Find 3 numbers a, b and c whose sum is 12 for which $ab + 2ac + 3bc$ is a maximum.

13. (15 points) Find the mass and center of mass of the conical **surface** $z = \sqrt{x^2 + y^2}$ for $z \leq 2$ with density $\rho = x^2 + y^2$. The cone may be parametrize as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.