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| 1-4 | /20 |
| 5 | / 5 |
| Total | /25 |

MATH 251 Quiz 2 Spring 2007

Sections 509 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

**1**. Find the equation of the line through the point $P = (4,3,2)$ in the direction $\vec{v} = (1,2,-1)$. Where does this line pass through the $xy$-plane?

**a**. $(2,-1,0)$

**b**. $(6,7,0)$ Correct Choice

**c**. $(2,1,0)$

**d**. $(6,-7,0)$

**e**. $\left(3,\frac{7}{2},0\right)$

The line is: $X = P + t\vec{v}$ or $(x,y,z) = (4+t, 3+2t, 2-t)$

This intersects the $xy$-plane when $z = 0$ or $2 - t = 0$ or $t = 2$.

So $x = 4 + t = 6$ and $y = 3 + 2t = 7$.

**2**. Find the equation of the plane through the point $P = (1,3,2)$ with normal $\vec{N} = (4,2,-2)$. Where does this plane pass through the $z$-axis?

**a**. $(0,0,-3)$ Correct Choice

**b**. $(0,0,-1)$

**c**. $(0,0,1)$

**d**. $(0,0,3)$

**e**. $(0,0,6)$

The line is: $\vec{N} \cdot X = \vec{N} \cdot P$ or $4x + 2y - 2z = 4 \cdot 1 + 2 \cdot 3 - 2 \cdot 2 = 6$

This intersects the $z$-axis when $x = y = 0$. So $-2z = 6$ or $z = -3$.

**3**. Classify the curve $x^2 - y^2 - 6x - 4y = -6$

**a**. circle with center $(3,-2)$

**b**. circle with center $(-3,2)$

**c**. hyperbola opening left and right

**d**. hyperbola opening up and down Correct Choice

**e**. parabola with vertex $(-3,2)$

Complete squares: $(x^2 - 6x + 9) - (y^2 + 4y + 4) = -6 + 9 - 4 = -1$

or $(x-3)^2 - (y+2)^2 = -1$ which is a hyperbola.

Since $(x-3)^2 + 1 = (y+2)^2$ we have $(y+2)^2 \geq 1$.

So the hyperbola opens up and down.

**4**. Classify the surface $x^2 + y^2 - 4x - 4y + z = -4$

**a**. hyperboloid of 1 sheet
**b**. hyperboloid of 2 sheets
**c**. hyperbolic paraboloid
**d**. elliptic paraboloid opening up
**e**. elliptic paraboloid opening down Correct Choice

Complete squares: $(x^2 - 4x + 4) + (y^2 + 4y + 4) + z = -4 + 4 + 4 = 4$
or $z = 4 - (x-2)^2 - (y-2)^2$ which is a paraboloid.
Since the coefficients of $x^2$ and $y^2$ are both negative, the paraboloid is elliptic, opening down.

**5**. Find the point where the line $\frac{x-1}{-1} = \frac{y-5}{2} = z - 6$ intersects the plane $3x - 2y + z = 11$.
HINT: Use the line to write $x$ and $y$ as functions of $z$.
Solve this on the back of the Scantron. Show all work.

$x - 1 = -1(z-6)$ $x = -z + 7$ $y - 5 = 2(z-6)$ $y = 2z - 7$

Plug into the plane:

$3(-z+7) - 2(2z-7) + z = 11$ or $-3z + 21 - 4z + 14 + z = 11$ or $-6z = -24$

So $z = 4$ $x = -z + 7 = -4 + 7 = 3$ $y = 2z - 7 = 2 \cdot 4 - 7 = 1$

So the point is: $\boxed{(x,y,z) = (3,1,4)}$

Check: $\frac{x-1}{-1} = \frac{3-1}{-1} = -2$ $\frac{y-5}{2} = \frac{1-5}{2} = -2$ $z - 6 = 4 - 6 = -2$ Good

$3x - 2y + z = 3 \cdot 3 - 2 \cdot 1 + 4 = 11$ Good