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MATH 251 Quiz 3 Spring 2007

Sections 509 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

1. Find the equation of the plane tangent to the surface $ze^{xy-2} = 3$ at the point $(2,1,3)$. Its z-intercept is:

- **a.** 3
- **b.** -3
- **c.** 15 Correct Choice
- **d.** -15
- **e.** 0

$P = (2,1,3)$ $F = ze^{xy-2}$ $\vec{\nabla}F = \langle yze^{xy-2}, xze^{xy-2}, e^{xy-2}\rangle$ $\vec{N} = \vec{\nabla}F\big|_P = \langle 3,6,1\rangle$

Tangent plane is $\vec{N}\cdot X = \vec{N}\cdot P$ or $3x + 6y + z = 3\cdot 2 + 6\cdot 1 + 1\cdot 3 = 15$

or $z = 15 - 3x - 6y$ The z-intercept is 15.

2. Find the equation of the line perpendicular to the surface $ze^{xy-2} = 3$ at the point $(2,1,3)$. It intersects the xy-plane at:

- **a.** $(7,17,0)$
- **b.** $(-7,-17,0)$ Correct Choice
- **c.** $(11,19,0)$
- **d.** $(-11,-19,0)$
- **e.** $(11,19,6)$

$P = (2,1,3)$ $F = ze^{xy-2}$ $\vec{\nabla}F = \langle yze^{xy-2}, xze^{xy-2}, e^{xy-2}\rangle$ $\vec{v} = \vec{\nabla}F\big|_P = \langle 3,6,1\rangle$

Normal line is $X = P + t\vec{v} = (2,1,3) + t\langle 3,6,1\rangle$ or $(x,y,z) = (2+3t, 1+6t, 3+t)$

The line intersects the xy-plane when $z = 0$ or $3 + t = 0$ or $t = -3$

$(x,y,z) = (2+3(-3), 1+6(-3), 3+(-3)) = (-7,-17,0)$.

3. If the temperature in a room is given by $T = 75 + xy^2z$ and a fly is located at $(2,1,3)$, in what **unit** vector direction should the fly fly in order to **decrease** the temperature as fast as possible?

a. $\langle 3,12,2 \rangle$

b. $\langle 3,-12,2 \rangle$

c. $\langle -3,-12,-2 \rangle$

d. $\dfrac{1}{\sqrt{157}}\langle 3,12,2 \rangle$

e. $\dfrac{1}{\sqrt{157}}\langle -3,-12,-2 \rangle$ Correct Choice

$\vec{\nabla} T = \langle y^2z, 2xyz, xy^2 \rangle \qquad \vec{v} = \vec{\nabla} T\Big|_{(2,1,3)} = \langle 3,12,2 \rangle \qquad |\vec{v}| = \sqrt{9+144+4} = \sqrt{157}$

Direction of Max increase is $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{1}{\sqrt{157}}\langle 3,12,2 \rangle$.

Direction of Max decrease is $-\hat{v} = \dfrac{-1}{\sqrt{157}}\langle 3,12,2 \rangle$.

4. Which of the following is NOT a critical point of $f(x,y) = (2x-x^2)(4y-y^2)$?

a. $(0,0)$
b. $(0,4)$
c. $(1,2)$
d. $(2,0)$
e. $(-2,4)$ Correct Choice

$f_x = (2-2x)(4y-y^2) = 0 \qquad f_y = (2x-x^2)(4-2y) = 0$

From $f_x = 0$, either $x = 1$ or $y = 0$ or $y = 4$

Case $x = 1$: From $f_y = 0$, $(4-2y) = 0 \Rightarrow y = 2$

Case $y = 0$: From $f_y = 0$, $(2x-x^2)4 = 0 \Rightarrow x = 0$ or $x = 2$

Case $y = 4$: From $f_y = 0$, $(2x-x^2)(-4) = 0 \Rightarrow x = 0$ or $x = 2$

The critical points are: $(1,2)$, $(0,0)$, $(2,0)$, $(0,4)$, $(2,4)$

OR Simply plug each answer into f_x and f_y

5. Find 3 numbers a, b and c whose sum is 80 for which $ab + 2bc + 3ac$ is a maximum.

Solve on the back of the Scantron.

We need to maximize $f = ab + 2bc + 3ac$ subject to the constraint $a + b + c = 80$.

$c = 80 - a - b \qquad f = ab + 2b(80-a-b) + 3a(80-a-b) = 240a + 160b - 3a^2 - 2b^2 - 4ab$

$f_a = 240 - 6a - 4b = 0 \qquad f_b = 160 - 4b - 4a = 0$

$6a + 4b = 240 \qquad 4a + 4b = 160$

Subtract: $2a = 80 \quad a = 40$ Substitute back: $4b = 160 - 4a = 0 \quad b = 0$

$c = 80 - a - b = 40$

So $\boxed{a = 40, \quad b = 0, \quad c = 40}$