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1-3	/15
4	/10
Total	/25

MATH 251 Quiz 4 Spring 2007

Sections 509 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. The point $(1,2)$ is a critical point of $f(x,y) = (2x - x^2)(4y - y^2)$.
Use the Second Derivative Test to classify $(1,2)$ as one of the following:

a. Local Minimum

b. Local Maximum Correct Choice

c. Inflection Point

d. Saddle Point

e. Test Fails

$f_x = (2-2x)(4y-y^2) = 0$ $\quad f_y = (2x-x^2)(4-2y) = 0$

$f_{xx} = -2(4y-y^2)$ $\quad f_{yy} = -2(2x-x^2)$ $\quad f_{xy} = (2-2x)(4-2y)$

$f_{xx}(1,2) = -8$ $\quad f_{yy}(1,2) = -2$ $\quad f_{xy}(1,2) = 0$ $\quad D = f_{xx}(1,2)f_{yy}(1,2) - f_{xy}(1,2)^2 = 16$

$D > 0 \quad \& \quad f_{xx} < 0 \quad \Rightarrow$ Local Maximum

2. Find the volume of the solid below the surface $z = xy$ above the region between the curves $y = x^2$, $y = 0$ and $x = 2$.

a. $\dfrac{64}{3}$

b. $\dfrac{32}{3}$

c. $\dfrac{16}{3}$ Correct Choice

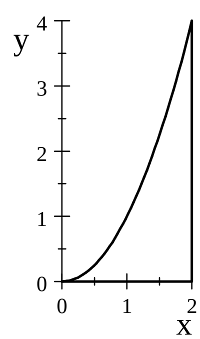
d. $\dfrac{8}{3}$

e. $\dfrac{4}{3}$

$$V = \int_0^2 \int_0^{x^2} xy\,dy\,dx = \int_0^2 x\left[\frac{y^2}{2}\right]_{y=0}^{x^2} dx$$

$$= \frac{1}{2}\int_0^2 x(x^4)\,dx = \frac{1}{2}\int_0^2 (x^5)\,dx$$

$$= \frac{1}{2}\left[\frac{x^6}{6}\right]_{x=0}^2 = \frac{1}{2}\left(\frac{2^6}{6}\right) = \frac{16}{3}$$



3. Reverse the order of integration in the integral $\int_0^4 \int_0^{\sqrt{y}} e^{x^3+y^4}\,dx\,dy$

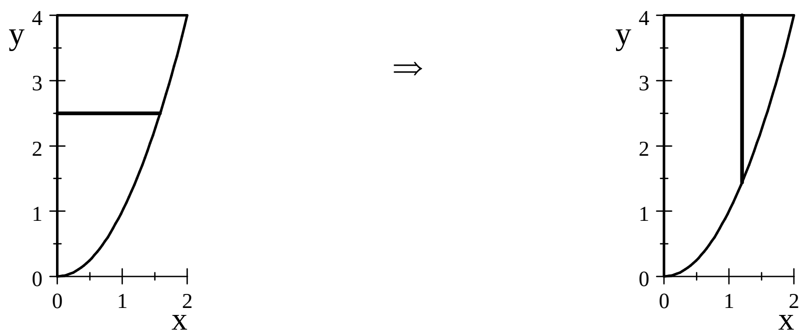
a. $\int_0^2 \int_{x^2}^4 e^{x^3+y^4}\,dy\,dx$ Correct Choice

b. $\int_0^{16} \int_0^{x^2} e^{x^3+y^4}\,dy\,dx$

c. $\int_0^2 \int_{x^2}^4 e^{x^4+y^3}\,dy\,dx$

d. $\int_0^{16} \int_0^{x^2} e^{x^4+y^3}\,dy\,dx$

e. $\int_0^2 \int_0^{x^2} e^{x^3+y^4}\,dy\,dx$



4. (10 points) Find the mass and x-component of the center of mass of the plate in the first quadrant bounded by $y = 3 - x$, the x-axis and the y-axis if the surface density is $\rho = y$.

Solve on the back of the Scantron.

This is a triangle.

$$M = \iint \rho\,dA = \int_0^3 \int_0^{3-x} y\,dy\,dx = \int_0^3 \left[\frac{y^2}{2} \right]_{y=0}^{3-x} dx = \int_0^3 \frac{(3-x)^2}{2}\,dx = \frac{1}{2}\int_0^3 (9 - 6x + x^2)\,dx$$

$$= \frac{1}{2}\left[9x - 3x^2 + \frac{x^3}{3} \right]_{x=0}^{3} = \frac{1}{2}(27 - 27 + 9) = \frac{9}{2}$$

$$x\text{-mom} = \iint x\rho\,dA = \int_0^3 \int_0^{3-x} xy\,dy\,dx = \int_0^3 \left[\frac{xy^2}{2} \right]_{y=0}^{3-x} dx = \int_0^3 \frac{x(3-x)^2}{2}\,dx = \frac{1}{2}\int_0^3 (9x - 6x^2 + x^3)\,dx$$

$$= \frac{1}{2}\left[9\frac{x^2}{2} - 2x^3 + \frac{x^4}{4} \right]_{x=0}^{3} = \frac{1}{2}\left(\frac{81}{2} - 54 + \frac{81}{4} \right) = \frac{27}{8}$$

$$\bar{x} = \frac{x\text{-mom}}{M} = \frac{27}{8}\frac{2}{9} = \frac{3}{4}$$