

## Multiple Choice: (5 points each)

1. Find the equation of the sphere which passes through the points  $(2, 1, 3)$  and  $(4, 5, -1)$ .

- a.  $(x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 3$
- b.  $(x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 9$     **Correct Choice**
- c.  $(x + 3)^2 + (y + 3)^2 + (z + 1)^2 = 9$
- d.  $(x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 36$
- e.  $(x + 3)^2 + (y + 3)^2 + (z + 1)^2 = 36$

The center is the midpoint:  $\vec{C} = \frac{P+Q}{2} = (3, 3, 1)$

The diameter is the distance:  $d(P, Q) = \sqrt{2^2 + 4^2 + 4^2} = 6$

The radius is half the diameter:  $r = 3$

The circle is:  $(x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 9$

2. The angle between the vectors  $\vec{u} = (2, -2, 4)$  and  $\vec{v} = (-1, 2, 1)$  is

- a. acute
- b. right
- c. obtuse    **Correct Choice**
- d. none of these

Since  $\vec{u} \cdot \vec{v} = -2 - 4 + 4 = -2 < 0$ , the angle is obtuse.

3. A wagon is pulled horizontally from the origin  $(0, 0)$  to the point  $(5, 0)$  meters by the force  $\vec{F} = (4, 3)$  Newtons. Find the work done.

- a. 20 Joules    **Correct Choice**
- b. 15 Joules
- c. 25 Joules
- d.  $5\sqrt{5}$  Joules
- e.  $\sqrt{5}$  Joules

The displacement vector is  $\vec{D} = (5, 0)$ . So the work is  $W = \vec{F} \cdot \vec{D} = 10$  Joules.

4. A triangle has vertices  $P = (-1, 2, -3)$ ,  $Q = (3, 2, 1)$ , and  $R = (-1, -1, 0)$ . Find the angle at  $P$ .

- a.  $30^\circ$
- b.  $45^\circ$
- c.  $60^\circ$     Correct Choice
- d.  $90^\circ$
- e.  $120^\circ$

$$\begin{aligned} \vec{PQ} &= Q - P = (4, 0, 4) & \vec{PR} &= R - P = (0, -3, 3) \\ |\vec{PQ}| &= \sqrt{16 + 16} = 4\sqrt{2} & |\vec{PR}| &= \sqrt{9 + 9} = 3\sqrt{2} & \vec{PQ} \cdot \vec{PR} &= 12 \\ \cos \theta &= \frac{12}{4\sqrt{2} \cdot 3\sqrt{2}} = \frac{1}{2} & \theta &= 60^\circ & & \text{(Use a 30-60-90 triangle.)} \end{aligned}$$

5. A triangle has vertices  $P = (-1, 2, -3)$ ,  $Q = (3, 2, 1)$ , and  $R = (-1, -1, 0)$ . Find the projection of the side  $\vec{PQ}$  onto the base  $\vec{PR}$ .

- a.  $\text{proj}_{\vec{PR}} \vec{PQ} = \left(\frac{3}{2}, 0, \frac{3}{2}\right)$
- b.  $\text{proj}_{\vec{PR}} \vec{PQ} = (-1, -1, 0)$
- c.  $\text{proj}_{\vec{PR}} \vec{PQ} = \left(-\frac{2}{3}, -\frac{2}{3}, 0\right)$
- d.  $\text{proj}_{\vec{PR}} \vec{PQ} = (0, -3, 3)$
- e.  $\text{proj}_{\vec{PR}} \vec{PQ} = (0, -2, 2)$     Correct Choice

$$\begin{aligned} \vec{PQ} &= Q - P = (4, 0, 4) & \vec{PR} &= R - P = (0, -3, 3) \\ |\vec{PR}|^2 &= 9 + 9 = 18 & \vec{PQ} \cdot \vec{PR} &= 12 \\ \text{proj}_{\vec{PR}} \vec{PQ} &= \frac{\vec{PQ} \cdot \vec{PR}}{|\vec{PR}|^2} \vec{PR} = \frac{12}{18} (0, -3, 3) = (0, -2, 2) \end{aligned}$$