Name__________________________ Sec______

MATH 251/253 Quiz 3 Spring 2008

Section 508/200,501,502 Solutions P. Yasskin

1	/10	3	/10
2	/ 5	Total	/25

1. (10 points) Find all critical points of the function $f = 2x^2y + 3xy^2 + 6xy$. Then use the 2nd Derivative Test to classify each as a local minimum, local maximum or saddle point or say the test fails.

Find Critical Points:

$f_x = 4xy + 3y^2 + 6y = y(4x + 3y + 6)$ $\quad f_y = 2x^2 + 6xy + 6x = x(2x + 6y + 6)$

$f_x = 0 \quad \Rightarrow \quad y = 0$ or $4x + 3y + 6 = 0$

$f_y = 0 \quad \Rightarrow \quad x = 0$ or $2x + 6y + 6 = 0$

Case 1: $y = 0$ and $x = 0$ $\qquad (0,0)$

Case 2: $y = 0$ and $2x + 6y + 6 = 0 \quad \Rightarrow \quad 2x + 6 = 0 \quad x = -3$ $\qquad (-3,0)$

Case 3: $4x + 3y + 6 = 0$ and $x = 0 \quad \Rightarrow \quad 3y + 6 = 0 \quad y = -2$ $\qquad (0,-2)$

Case 4: (1) $4x + 3y + 6 = 0$ and (2) $2x + 6y + 6 = 0$

$2 \times$ (1) - (2): $6x + 6 = 0 \quad x = -1$

(1): $-4 + 3y + 6 = 0 \quad y = -2/3$ $\qquad (-1,-2/3)$

Classify:

$f_{xx} = 4y \quad f_{yy} = 6x \quad f_{xy} = 4x + 6y + 6 \quad D = f_{xx}f_{yy} - f_{xy}^2$

x	y	f_{xx}	f_{yy}	f_{xy}	D	Type
0	0	0	0	6	–36	saddle
–3	0	0	–18	–6	–36	saddle
0	–2	–8	0	–6	–36	saddle
–1	–2/3	–8/3	–6	–2	12	local maximum

2. (5 points) If the temperature in a room is given by $T = 75 + xy + xz + yz$. Find the rate of change of the temperature **in the direction of** the vector $(12, 4, 3)$ at the point $(1, 0, 2)$.

$$\vec{\nabla}T = (y+z, x+z, x+y) \qquad \vec{\nabla}T\Big|_{(1,0,2)} = (2, 3, 1)$$

$$\vec{v} = (12, 4, 3) \qquad |\vec{v}| = \sqrt{144+16+9} = 13 \qquad \hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{13}(12, 4, 3)$$

$$D_{\hat{v}}f = \hat{v} \cdot \vec{\nabla}T\Big|_{(1,0,2)} = \frac{1}{13}(12(2) + 4(3) + 3(1)) = 3$$

3. (10 points) A rectangular box sits on the xy-plane with its upper vertices on the elliptic paraboloid $z = 36 - 9x^2 - 4y^2$. Find the **dimensions** and **volume** of the largest such box.

Method of Eliminating a Variable:

Maximize $V = (2x)(2y)z = 4xyz$ subject to $z = 36 - 9x^2 - 4y^2$.

Maximize $V = 4xy(36 - 9x^2 - 4y^2) = 144xy - 36x^3y - 16xy^3$.

$V_x = 144y - 108x^2y - 16y^3 = 4y(36 - 27x^2 - 4y^2) = 0$

$V_y = 144x - 36x^3 - 48xy^2 = 4x(36 - 9x^2 - 12y^2) = 0$

To have a non-zero volume, we must have $x \neq 0$ and $y \neq 0$.

So we must solve $\left\{ \begin{array}{ll} 36 - 27x^2 - 4y^2 = 0 & \text{eq 1} \\ 36 - 9x^2 - 12y^2 = 0 & \text{eq 2} \end{array} \right\}$

$3 \times (\text{eq 1}) - (\text{eq 2})$ is: $72 - 72x^2 = 0$ or $x^2 = 1$ or $x = 1$

Substitute into eq 1: $36 - 27 - 4y^2 = 0$ or $4y^2 = 9$ or $y = \frac{3}{2}$

Substitute back: $z = 36 - 9x^2 - 4y^2 = 36 - 9 - 9 = 18$

The dimensions are: $2 \times 3 \times 18$

The volume is: $V = 4xyz = 4 \cdot 1 \cdot \frac{3}{2} \cdot 18 = 108$

Method of Lagrange Multipliers:

Maximize $V = (2x)(2y)z = 4xyz$ subject to $g = z + 9x^2 + 4y^2 = 36$.

$\vec{\nabla}V = (4yz, 4xz, 4xy)$ $\quad \vec{\nabla}g = (18x, 8y, 1)$ $\quad$ Lagrange equations: $\vec{\nabla}V = \lambda\vec{\nabla}g$:

$4yz = \lambda 18x, \quad 4xz = \lambda 8y, \quad 4xy = \lambda$

$\frac{4xyz}{\lambda} = 18x^2 = 8y^2 = z$

From the constraint: $36 = z + \frac{z}{2} + \frac{z}{2} = 2z \qquad z = 18$

$18x^2 = 8y^2 = z = 18 \qquad x = 1 \qquad y = \frac{3}{2}$

The dimensions are: $2 \times 3 \times 18$

The volume is: $V = 4xyz = 4 \cdot 1 \cdot \frac{3}{2} \cdot 18 = 108$