

Name _____ Sec _____

MATH 251 Exam 2 Spring 2011

Section 511 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1-12	/60	14	/15
13	/15	15	/15
		Total	/105

1. Compute $\int_0^2 \int_0^y xy \, dx \, dy$.

- a. 1
- b. 2
- c. 3
- d. 4
- e. y^2

2. Find the area of one loop of the rose $r = \sin(3\theta)$.

- a. $\frac{\pi}{12}$
- b. $\frac{\pi}{6}$
- c. $\frac{\pi}{4}$
- d. $\frac{\pi}{3}$
- e. $\frac{\pi}{2}$

3. Compute $\iiint x^2 + y^2 \, dV$ over the region between the cones $z = \sqrt{x^2 + y^2}$ and $z = 4 - \sqrt{x^2 + y^2}$.

- a. $\frac{8\pi}{3}$
- b. $\frac{16\pi}{3}$
- c. $\frac{32\pi}{3}$
- d. $\frac{16\pi}{5}$
- e. $\frac{32\pi}{5}$

4. Find the mass of the hemisphere $x^2 + y^2 + z^2 \leq 4$ with $y \geq 0$ if the density is $\delta = y$.

- a. $\frac{\pi}{2}$
- b. π
- c. 2π
- d. 4π
- e. 8

5. Find the center of mass of the hemisphere $x^2 + y^2 + z^2 \leq 4$ with $y \geq 0$ if the density is $\delta = y$.

- a. $(0, \frac{64\pi}{15}, 0)$
- b. $(0, \frac{16}{15}, 0)$
- c. $(0, \frac{\pi^2}{12}, 0)$
- d. $(0, \frac{15}{16}, 0)$
- e. $(0, \frac{12}{\pi^2}, 0)$

6. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-16x^2y, 9xy^2)$ counterclockwise around the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

HINTS: The ellipse may be parametrized by $\vec{r}(\theta) = (3 \cos \theta, 4 \sin \theta)$.

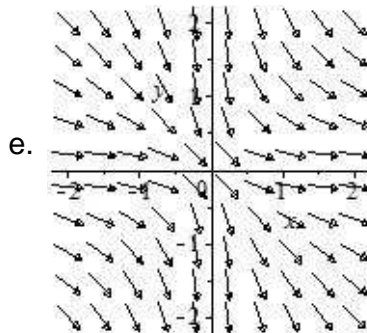
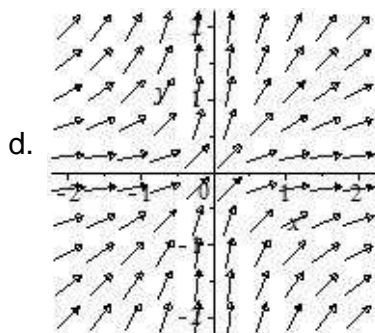
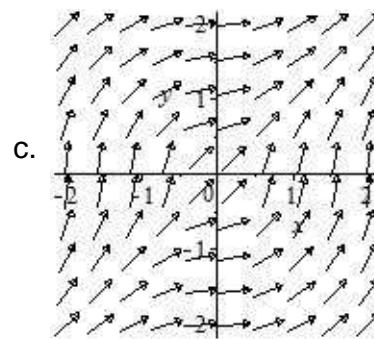
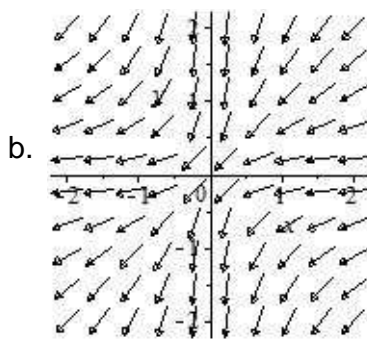
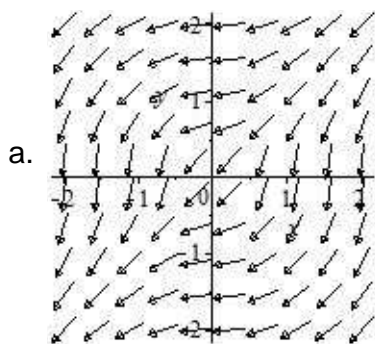
Since $\sin(2\theta) = 2 \sin \theta \cos \theta$, we have $4 \sin^2 \theta \cos^2 \theta = \sin^2(2\theta)$.

- a. -864π
- b. -288π
- c. 144π
- d. 288π
- e. 864π

7. The point $\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$ is a critical point of the function $f(x,y) = \sin(x)\cos(y) - \frac{\sqrt{3}}{4}x + \frac{\sqrt{3}}{4}y$. Use the Second Derivative Test to classify this critical point.

- a. Local Maximum
- b. Local Minimum
- c. Inflection Point
- d. Saddle Point
- e. Test Fails

8. Which of the following is the plot of the vector field $\vec{F}(x,y) = \frac{1}{\sqrt{x^2 + y^2}} (|x|, |y|)$?



9. Find the area of the parametric surface $\vec{R}(u,t) = (ue^t, ue^{-t}, \sqrt{2}u)$ for $0 \leq u \leq 2$ and $0 \leq t \leq 1$.

HINT: Look for a perfect square.

- a. $2\sqrt{2}\left(e + \frac{1}{e} - 2\right)$
- b. $2\sqrt{2}\left(e + \frac{1}{e}\right)$
- c. $2\sqrt{2}\left(e - \frac{1}{e} - 2\right)$
- d. $2\sqrt{2}\left(e - \frac{1}{e}\right)$
- e. $2\left(e + \frac{1}{e}\right)$

10. Find the equation of the plane tangent to the parametric surface $\vec{R}(u,t) = (ue^t, ue^{-t}, \sqrt{2}u)$ at the point $P = \vec{R}(2,0)$ where $u = 2$ and $t = 0$.

Hint: Evaluate the normal \vec{N} at $u = 2$ and $t = 0$.

- a. $x + y - \sqrt{2}z = -4\sqrt{2}$
- b. $x + y - \sqrt{2}z = 0$
- c. $x + y - \sqrt{2}z = 16\sqrt{2}$
- d. $\sqrt{2}x - \sqrt{2}y + 2z = -4\sqrt{2}$
- e. $\sqrt{2}x - \sqrt{2}y + 2z = 0$

11. If $\vec{F} = (xy \tan z, yz \cos x, xz \sin y)$, then $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} =$

- a. $-2z \cos y - 2x(\tan^2 z + 1)$
- b. $-2z \cos y + 2x(\tan^2 z + 1)$
- c. $2z \cos y - 2x(\tan^2 z + 1)$
- d. $2z \cos y + 2x(\tan^2 z + 1)$
- e. 0

12. Let f be the scalar potential for $\vec{F} = (2xz - 3y, 8yz - 3x, x^2 + 4y^2 + 2z)$ for which $f(0, 0, 0) = 0$. Then $f(1, 1, 1) =$

- a. 1
- b. 2
- c. 3
- d. 4
- e. 5

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (15 points) The plane $x + 2y + 4z = 8$ intersects the 1st octant ($x > 0, y > 0, z > 0$) in a triangle. Find the point on this triangle at which the function $f = xy^2z^3$ is a maximum.

14. (15 points) Compute $\iiint \vec{\nabla} \cdot \vec{F} dV$ for $\vec{F} = (xy^2, yz^2, zx^2)$ over the solid above the cone $z = \sqrt{x^2 + y^2}$ below the sphere $x^2 + y^2 + z^2 = 4$.

15. (15 points) Compute $\iint_E \vec{\nabla} \times \vec{F} \cdot \hat{k} dx dy$ for $\vec{F} = (-16x^2y, 9xy^2, 0)$ over the interior of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

HINTS: First compute $\vec{\nabla} \times \vec{F} \cdot \hat{k}$ in rectangular coordinates.

Then compute the integral in elliptic coordinates $x = 3u \cos \theta$, $y = 4u \sin \theta$.