

Name _____ ID _____

MATH 251

Exam 1

Fall 2012

Sections 515

P. Yasskin

1-12	/60
13	/10
14	/10
15	/10
16	/10
Total	/100

Multiple Choice: (5 points each. No part credit.)

1. Find the area of the triangle whose vertices are

$$P = (2, 4, -3), \quad Q = (3, 4, -2) \quad \text{and} \quad R = (0, 6, -3).$$

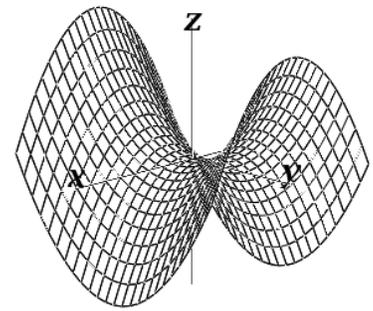
- a. 1
 - b. $\sqrt{3}$
 - c. $2\sqrt{3}$
 - d. 6
 - e. 12
2. Which of the following is the plane which passes through the point $(4, 2, 1)$ and is perpendicular to the line $(x, y, z) = (1 + 2t, 2 + t, 3 + 3t)$?
- a. $2x - y + 3z = 13$
 - b. $2x - y + 3z = 9$
 - c. $2x + y + 3z = 13$
 - d. $2x + y + 3z = 9$
 - e. $-2x + y - 3z = -9$

3. The quadratic surface $x^2 + y^2 - z^2 + 4x + 4y - 6z = 0$ is

- a. an elliptic hyperboloid
- b. a hyperbolic paraboloid
- c. a hyperboloid of 1 sheet
- d. a hyperboloid of 2 sheets
- e. a cone.

4. The plot at the right is the graph of which equation?

- a. $z = -x^2 + y^2$
- b. $z = x^2 - y^2$
- c. $z^2 = x^2 - y^2$
- d. $z^2 = -x^2 + y^2$
- e. $z^2 - x^2 - y^2 = 1$



5. An airplane is travelling due South with constant speed and constant altitude as it flies over College Station. Since its path is part of a circle around the earth, its acceleration points directly toward the center of the earth. In which direction does it binormal \hat{B} point?

- a. Up
- b. North
- c. East
- d. South
- e. West

6. For the curve $\vec{r}(t) = (4 \cos t, 3t, 4 \sin t)$ which of the following is FALSE?

a. $\vec{v} = \langle -4 \sin t, 3, 4 \cos t \rangle$

b. $\vec{a} = \langle -4 \cos t, 0, -4 \sin t \rangle$

c. $|\vec{v}| = 25$

d. Arc length between $t = 0$ and $t = 2\pi$ is 10π

e. $a_T = 0$

7. A wire in the shape of the curve $\vec{r}(t) = (4 \cos t, 3t, 4 \sin t)$ has linear mass density $\rho = y + z$. Find its total mass between $t = 0$ and $t = 2\pi$.

a. 6π

b. 12π

c. 30π

d. $6\pi^2$

e. $30\pi^2$

8. Find the work done to move an object along the curve $\vec{r}(t) = (4 \cos t, 3t, 4 \sin t)$ between $t = 0$ and $t = 2\pi$ by the force $\vec{F} = \langle z, 0, -x \rangle$?

a. -32π

b. -25π

c. $-25\pi^2$

d. 25π

e. 32π

9. Find the plane tangent to the graph of $z = xe^y$ at the point $(2,0)$. Its z -intercept is

- a. e
- b. 2
- c. 0
- d. -2
- e. $-e$

10. Find the plane tangent to the graph of $xz^3 + zy^2 + yx^4 = 42$ at the point $(1,2,0)$. Its z -intercept is

- a. 10
- b. $\frac{5}{4}$
- c. $\frac{5}{2}$
- d. $\frac{2}{5}$
- e. $\frac{4}{5}$

11. Hans Duo is currently at $(x, y, z) = (3, 2, 1)$ and flying the Milenium Eagle through a deadly polaron field whose density is $\rho = x^2z + yz^2$. In what unit vector direction should he travel to reduce the density as fast as possible?

- a. $\langle 6, 1, 13 \rangle$
- b. $\frac{1}{\sqrt{206}} \langle -6, 1, -13 \rangle$
- c. $\langle -6, -1, -13 \rangle$
- d. $\frac{1}{\sqrt{206}} \langle -6, -1, -13 \rangle$
- e. $\frac{1}{\sqrt{206}} \langle 6, -1, 13 \rangle$

12. The point $(x, y) = (9, 3)$ is a critical point of the function $f(x, y) = x^2 - 2xy^2 + 4y^3$. Use the Second Derivative Test to classify this critical point.

- a. local minimum
- b. local maximum
- c. saddle point
- d. TEST FAILS

Work Out: (10 points each. Part credit possible. Show all work.)

13. Find the scalar and vector projections of the vector $\vec{a} = \langle 1, 2, -2 \rangle$ along the vector $\vec{b} = \langle 2, -1, 2 \rangle$.

14. The pressure, P , volume, V , and temperature, T , of an ideal gas are related by

$$P = \frac{kT}{V} \quad \text{for some constant } k. \quad \text{For a certain sample } k = 10 \frac{\text{cm}^3 \cdot \text{atm}}{\text{K}}.$$

At a certain instant, the volume and temperature are $V = 2000 \text{ cm}^3$, and $T = 300 \text{ K}$, and are increasing at $\frac{dV}{dt} = 40 \frac{\text{cm}^3}{\text{sec}}$, and $\frac{dT}{dt} = 5 \frac{\text{K}}{\text{sec}}$.

At that instant, what is the pressure, is it increasing or decreasing and at what rate?

15. If two resistors, with resistances R_1 and R_2 , are arranged in parallel, the total resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

If $R_1 = 4\Omega$ and $R_2 = 6\Omega$ and the uncertainty in the measurement of R_1 is $\Delta R_1 = 0.03\Omega$ and for R_2 is $\Delta R_2 = 0.02\Omega$. Find R and use differentials to estimate the uncertainty in the measurement of R .

16. Find the point(s) on the surface $z^2 = 46 - 2x - 4y$ which are closest to the origin.
HINT: Explain why you can minimize the square of the distance instead of the distance.
Use the Second Derivative Test to check it is a local minimum.