1-9	/45
10	/15
11	/25
12	/15
Total	/100

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MATH 251 Final Exam Fall 2012

Sections 515 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Points A, B, C and D are the vertices of a parallelogram traversed in order.
If $A = (2,4,1)$, $B = (3,-2,0)$ and $D = (1,3,-2)$, then $C =$

a. $(2,-3,-3)$

b. $(4,-1,3)$

c. $(0,9,-1)$

d. $(6,5,-1)$

e. $\left(4,\frac{9}{2},0\right)$

2. Which vector is perpendicular to the surface $x^2z^3 + y^3z^2 = 1$ at the point $(3,-2,1)$?

a. $(12,-24,43)$

b. $(6,-12,43)$

c. $(6,12,43)$

d. $(6,-12,11)$

e. $(-12,-24,-22)$

3. Find the point on the elliptic paraboloid $\vec{R}(t,\theta) = (3t\cos\theta, 2t\sin\theta, 1+t^2)$ where a unit normal is $\hat{N} = \left(\frac{-2\sqrt{3}}{5}, \frac{-2}{5}, \frac{3}{5}\right)$.

a. $\left(\frac{3}{2}, \sqrt{3}, 2\right)$
b. $\left(-\frac{3}{2}\sqrt{3}, -1, 2\right)$
c. $\left(3, 2\sqrt{3}, 5\right)$
d. $\left(3\sqrt{3}, 2, 5\right)$
e. $\left(-3, -2\sqrt{3}, 5\right)$

4. Compute $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$

HINT: Plot the region of integration and convert to polar coordinates.

a. $\frac{\pi}{2}\ln 10$
b. $\pi \ln 10$
c. $\frac{\pi}{2}\arctan 3$
d. $\pi \arctan 3$
e. $\pi \arctan 10$

5. Find the mass of a wire in the shape of the curve $\vec{r}(t) = \left(e^t, \sqrt{2}\,t, e^{-t}\right)$ for $-1 \le t \le 1$ if the density is $\rho = x$.

a. $\frac{e^2}{2} + \frac{e^{-2}}{2}$

b. $\frac{e^2}{2} - \frac{e^{-2}}{2}$

c. $\frac{e^2}{2} - \frac{e^{-2}}{2} + 2$

d. $e^2 - e^{-2}$

e. $e^2 - e^{-2} + 2$

6. Find the plane tangent to graph of $z = x\cos y + \sin y$ at $(2, \pi)$.
What is the z-intercept?

a. $-4 + \pi$

b. $4 + \pi$

c. $-4 - \pi$

d. $4 - \pi$

e. π

7. Find the plane perpendicular to the curve $\vec{r}(t) = (t, t^2, t^3)$ at the point $(1,1,1)$.
What is the z-intercept?

a. 5
b. 4
c. 3
d. 2
e. 1

8. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (y^2, 4xy)$ along the piece of the parabola $y = x^2$ from $(-2,4)$ to $(2,4)$ followed by the line segment from $(2,4)$ back to $(-2,4)$.
HINT: Use Green's Theorem.

a. $\frac{256}{5}$
b. $\frac{768}{5}$
c. $\frac{64}{3}$
d. $\frac{128}{3}$
e. 0

9. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (4xy^2, 4x^2y)$ along the line segment from $(1,2)$ to $(3,1)$.
HINT: Find a scalar potential.

a. 4
b. 10
c. 20
d. 24
e. 26

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (15 points) A 166 cm piece of wire is cut into 3 pieces of lengths a, b and c.

The piece of length a is folded into a square of side $s = \frac{a}{4}$.

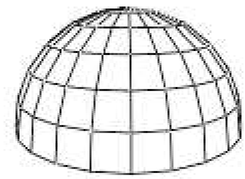
The piece of length b is folded into a rectangle of length $L_1 = \frac{b}{3}$ and width $W_1 = \frac{b}{6}$.

The piece of length c is folded into a rectangle of length $L_2 = \frac{3c}{8}$ and width $W_2 = \frac{c}{8}$.

Find a, b and c so that the total area is a minimum.

What is the total area?

11. (25 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (xz, yz, x^2 + y^2)$ and the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $\qquad\qquad\qquad\qquad$ $dV =$

c. Compute the left hand side:

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

Second the Right Hand Side:

The boundary surface consists of a hemisphere H and a disk D with appropriate orientations.

d. Parametrize the disk D:

$\vec{R}(r,\theta) = \big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\big)$

e. Compute the tangent vectors:

$\vec{e}_r = \big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\big)$

$\vec{e}_\theta = \big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\big)$

f. Compute the normal vector:

$\vec{N} =$

g. Evaluate $\vec{F} = (xz, yz, x^2 + y^2)$ on the disk:

$\vec{F}\big|_{\vec{R}(r,\theta)} =$

h. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

i. Compute the flux through D:

$\iint\limits_D \vec{F} \cdot d\vec{S} =$

j. Parametrize the hemisphere H:

$\vec{R}(\varphi,\theta) = \Big(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\Big)$

k. Compute the tangent vectors:

$\vec{e}_\varphi = \Big(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\Big)$

$\vec{e}_\theta = \Big(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\Big)$

l. Compute the normal vector:

$\vec{N} =$

m. Evaluate $\vec{F} = (xz, yz, x^2 + y^2)$ on the hemisphere:

$\vec{F}\Big|_{\vec{R}(\theta,\varphi)} =$

n. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

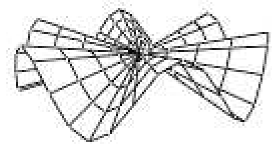
o. Compute the flux through H:

$\iint\limits_{C} \vec{F} \cdot d\vec{S} =$

p. Compute the **TOTAL** right hand side:

$\iint\limits_{\partial V} \vec{F} \cdot d\vec{S} =$

12. (15 points) Compute $\int\int \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-y, x, z)$



over the "clam shell" surface parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r\sin(6\theta))$$

for $r \leq 2$ oriented upward.

HINTS: Use Stokes Theorem.

What is the value of r on the boundary?