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1-13	/52
14	/12
15	/28
16	/12
Total	/104

MATH 251 Exam 2 Version A Spring 2013

Sections 506 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. Compute $\iint\limits_R xy\,dA$ over the region R between the parabola $y = x^2$ and the line $y = 2x$.

a. $\frac{4}{3}$

b. $\frac{8}{3}$

c. $\frac{32}{15}$

d. $\frac{10}{3}$

e. $\frac{29}{6}$

2. Compute $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x\,dy\,dx$ by converting to polar coordinates.

a. 36

b. 27

c. 18

d. 9

e. 3

3. Find the mass of the triangular plate with vertices $(0,0)$, $(2,-6)$ and $(2,6)$ if the surface density is $\rho = x^2$.

a. 4

b. 6

c. 12

d. 16

e. 24

4. Find the center of mass of the triangular plate with vertices $(0,0)$, $(2,-6)$ and $(2,6)$ if the surface density is $\rho = x^2$.

a. $\left(\frac{8}{5},0\right)$

b. $\left(\frac{9}{5},0\right)$

c. $\left(\frac{5}{8},0\right)$

d. $\left(\frac{5}{9},0\right)$

e. $\left(\frac{6}{5},0\right)$

5. Which of the following integrals is NOT equivalent to $\int_0^4 \int_0^{4-z} \int_0^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$? The region is shown.

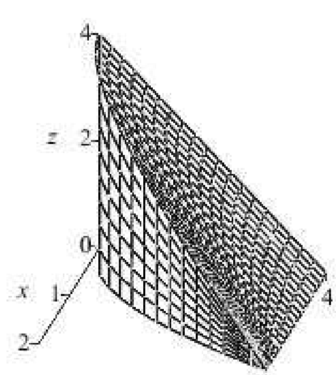
a. $\int_0^4 \int_0^{\sqrt{4-z}} \int_{x^2}^{4-z} f(x,y,z)\,dy\,dx\,dz$

b. $\int_0^4 \int_0^{4-y} \int_{y^2}^{2} f(x,y,z)\,dx\,dz\,dy$

c. $\int_0^4 \int_0^{\sqrt{y}} \int_0^{4-y} f(x,y,z)\,dz\,dx\,dy$

d. $\int_0^2 \int_0^{4-x^2} \int_{x^2}^{4-z} f(x,y,z)\,dy\,dz\,dx$

e. $\int_0^2 \int_{x^2}^{4} \int_0^{4-y} f(x,y,z)\,dz\,dy\,dx$



6. Find the area of one petal of the 8 petaled daisy $r = \sin(4\theta)$.

a. $\frac{\pi}{2}$

b. $\frac{\pi}{4}$

c. $\frac{\pi}{8}$

d. $\frac{\pi}{16}$

e. $\frac{\pi}{32}$

7. Find the mass of the solid between the paraboloids $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$ if the volume density is $\rho = z$.

a. 64π

b. 32π

c. 16π

d. 8π

e. 4π

8. Compute $\iiint (x^2 + y^2)\, z\, dV$ over the solid hemisphere $0 \le \sqrt{x^2 + y^2 + z^2} \le 2$

a. 0

b. $\frac{8}{3}\pi$

c. $\frac{16}{3}\pi$

d. $\frac{64}{9}\pi$

e. 2π

9. Which integral gives the arclength of the ellipse $\vec{r}(\theta) = (6\cos\theta, 3\sin\theta, 3\sin\theta)$?

a. $\int_0^{2\pi} 3\sqrt{2 + 2\sin^2\theta}\, d\theta$

b. $\int_0^{2\pi} 3\sqrt{4 + 2\cos^2\theta}\, d\theta$

c. $\int_0^{2\pi} 3\sqrt{2 + 2\cos^2\theta}\, d\theta$

d. $\int_0^{2\pi} \sqrt{2}\,(3 + 3\sin^2\theta)\, d\theta$

e. $\int_0^{2\pi} \sqrt{54}\, d\theta$

10. A helical thermocouple whose shape is the curve $\vec{r}(t) = (3\cos t, 3\sin t, 4t)$ for $0 \le t \le 4\pi$ is placed in a pot of water where the temperature is $T = (41 + x^2 + y^2 + z)$°C. Find the average temperature of the water as measured by the thermocouple.

HINT: $f_{\text{ave}} = \dfrac{\int f ds}{\int ds}$

a. $\dfrac{173}{4} + 8\pi$

b. $50 + 16\pi$

c. $1000\pi + 160\pi^2$

d. $250 + 40\pi$

e. $50 + 8\pi$

11. Find a scalar potential, $f(x,y,z)$, for $\vec{F}(x,y,z) = (2xy^2 + 2x + 2xz, 2x^2y - 3z, x^2 + 3z^2 - 3y)$.
Then compute $f(2,2,2) - f(1,1,1)$.

a. 0

b. 1

c. 7

d. 23

e. 25

12. If $f = x^2 + y^2 - 2z^2$ and $\vec{F} = (xz, yz, -z^2)$, which of the following is false?

a. $\vec{\nabla} \times \vec{\nabla} f = \vec{0}$

b. $\vec{\nabla} \cdot \vec{\nabla} f = 0$

c. $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} = 0$

d. $\vec{\nabla}\left(\vec{\nabla} \cdot \vec{F}\right) = \vec{0}$

e. None of the above. They are all true.

13. Compute $\iiint_C \vec{\nabla} \cdot \vec{G}\, dV$ for $\vec{G} = (xz, yz, z^2)$ over the solid cylinder $x^2 + y^2 \le 25$ with $0 \le z \le 4$.

a. 800π

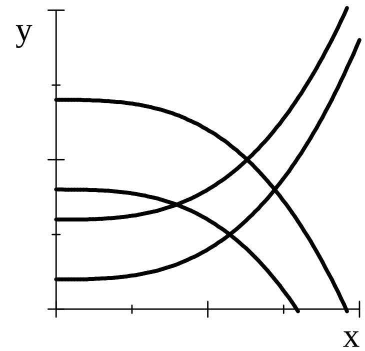
b. 400π

c. 200π

d. 80π

e. 40π

Work Out: (Points indicated. Part credit possible. Show all work.)



14. (12 points) Compute $\iint_D x^2\,dA$ over the "diamond" shaped region bounded by the curves

$y = 1 + x^3$, $y = 3 + x^3$, $y = 4 - x^3$, $y = 7 - x^3$.

HINT: Define curvilinear coordinates (u,v) so that $y = u + x^3$ and $y = v - x^3$.

a. (2 pts) What are the boundaries in terms of u and v?

b. (3 pts) Find formulas for x and y in terms of u and v.

c. (4 pts) Find the Jacobian factor $J = \left|\dfrac{\partial(x,y)}{\partial(u,v)}\right|$.

d. (1 pts) Express the integrand in terms of u and v.

e. (2 pts) Compute the integral.

15. (28 points) Consider the elliptical region, E, in the plane $z = 2 + x + y$ above the circle $x^2 + y^2 \le 4$ oriented upwards.

HINT: This ellipse may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2 + r\cos\theta + r\sin\theta)$.

a. (10 pts) Find the normal vector $\vec{N}$ to the ellipse and its length $|\vec{N}|$.

Note: $\vec{N}$ starts hard but simplifies!

b. (3 pts) Find the surface area of the ellipse.

c. (3 pts) Find the mass of the ellipse if the surface density is $\rho = x^2 + y^2$.

d. (12 pts) If $\vec{F} = (-yz, xz, z^2)$, compute the surface integral $\iint_E \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

16. (12 points) Compute $\iint_C \vec{G} \cdot d\vec{S}$ for $\vec{G} = (xz, yz, z^2)$ over the cylinder $x^2 + y^2 = 25$ for $0 \le z \le 4$ with outward normal.