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MATH 251/253 (circle one) Exam 1 Fall 2014

Sections 508/201/202(circle one) P. Yasskin

1-14	/70
15	/10
16	/10
17	/10
Total	/100

Multiple Choice: (5 points each. No part credit.)

1. The vertices of a triangle are $A = \left(2, 1, \sqrt{2}\right)$, $B = \left(3, 2, 2\sqrt{2}\right)$ and $C = \left(4, 3, \sqrt{2}\right)$.
Find the angle at A.

a. $30°$
b. $45°$
c. $60°$
d. $120°$
e. $135°$

2. The vertices of a triangle are $A = \left(2, 1, \sqrt{2}\right)$, $B = \left(3, 2, 2\sqrt{2}\right)$ and $C = \left(4, 3, \sqrt{2}\right)$.
Find a vector perpendicular to the plane of this triangle.

a. $(1, -1, 0)$
b. $(1, 1, 0)$
c. $(1, -1, 1)$
d. $(1, 1, 1)$
e. $(-1, -1, 1)$

3. Which of the following points lies on the line $(x, y, z) = (2 - t, 3 + 2t, 4 + t)$ and on the plane $2x + 3y + 4z = 21$?

a. $(1, 1, 1)$
b. $(4, 3, 2)$
c. $(2, 3, 2)$
d. $(3, 1, 3)$
e. $(2, 2, 2)$

4. The quadratic surface $x^2 - y^2 - 6x + 4y + 2 = 0$ is a

a. hyperboloid
b. hyperbolic ellipsoid
c. hyperbola
d. hyperboic paraboloid
e. hyperbolic cylinder

5. For the "twisted cubic" curve $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$, find the binormal vector $\hat{B}$.

a. $\left(\frac{2t^2}{2t^2+1}, \frac{-2t}{2t^2+1}, \frac{1}{2t^2+1}\right)$
b. $\left(\frac{1}{2t^2+1}, \frac{2t}{2t^2+1}, \frac{2t^2}{2t^2+1}\right)$
c. $\left(\frac{-2t^2}{2t^2+1}, \frac{2t}{2t^2+1}, \frac{-1}{2t^2+1}\right)$
d. $\left(\frac{1}{2t^2+1}, \frac{-2t}{2t^2+1}, \frac{2t^2}{2t^2+1}\right)$
e. $\left(\frac{2t^2}{2t^2+1}, \frac{2t}{2t^2+1}, \frac{1}{2t^2+1}\right)$

6. Find the mass of the "twisted cubic" curve $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ between $t = 0$ and $t = 1$ if the linear density is $\rho = y^2 + 6xz$.

- **a.** 1
- **b.** $\frac{1}{5}$
- **c.** $\frac{7}{5}$
- **d.** $\frac{20}{7}$
- **e.** $\frac{17}{7}$

7. Find the work done when a bead is pushed along the "twisted cubic" curve $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ between $t = 0$ and $t = 1$ if you apply the force $\vec{F} = (3z, y, x)$.

- **a.** $\frac{1}{2}$
- **b.** 1
- **c.** $\frac{3}{2}$
- **d.** 2
- **e.** $\frac{5}{2}$

8. You are riding on a train which is currently travelling EAST but curving toward the SOUTH. Where do $\hat{B}$ and $\hat{N}$ for the train currently point?

- **a.** $\hat{B}$ points SOUTH and $\hat{N}$ points DOWN.
- **b.** $\hat{B}$ points SOUTH and $\hat{N}$ points UP.
- **c.** $\hat{B}$ points UP and $\hat{N}$ points SOUTH.
- **d.** $\hat{B}$ points DOWN and $\hat{N}$ points SOUTH.
- **e.** $\hat{B}$ points DOWN and $\hat{N}$ points SOUTHEAST.

9. For the function $f = x\sin(yz)$, which of the following are correct?

I. $\dfrac{\partial^2 f}{\partial x \partial y} = -z\cos yz$ III. $\dfrac{\partial^2 f}{\partial x \partial z} = y\cos yz$ V. $\dfrac{\partial^2 f}{\partial y \partial z} = x\cos yz - xyz\sin yz$

II. $\dfrac{\partial^2 f}{\partial y \partial x} = z\cos yz$ IV. $\dfrac{\partial^2 f}{\partial z \partial x} = y\cos yz$ VI. $\dfrac{\partial^2 f}{\partial z \partial y} = x\cos yz + xyz\sin yz$

a. I and II.
b. III and IV.
c. V and VI.
d. I, II and III.
e. IV, V and VI.

10. Find the equation of the plane tangent to the graph of the function $z = f(x,y) = x^2 y + xy^3$ at $(x,y) = (2,1)$. What is the z-intercept?

a. -14
b. -6
c. 6
d. 14
e. 26

11. Find the equation of the plane tangent to the graph of the equation $x\sin(yz) = 1$ at $P = \left(\sqrt{2}, \frac{1}{4}, \pi\right)$. What is the z-intercept?

a. $\sqrt{2} + \frac{\pi}{4}$
b. $1 + \frac{\pi}{2}$
c. $2 + \pi$
d. $4 + 2\pi$
e. $2\sqrt{2} + 2\pi$

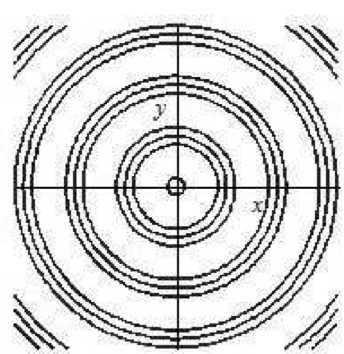
12. A fish is currently at the point $(x,y,z) = (1,2,-3)$ and has velocity $\vec{v} = (1,2,1)$. If the salt density is $D = xyz^2$, find $\dfrac{dD}{dt}$, the time rate of change of the density as seen by the fish at the current instant.

a. 12
b. 24
c. 36
d. 48
e. 60

13. The equation $z^3 \sin x + z\cos y = 3$ defines z as an implicit function of x and y. Notice that its graph passes through the point $\left(\frac{\pi}{4}, \frac{\pi}{4}, \sqrt{2}\right)$. Find $\dfrac{\partial z}{\partial y}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

a. $\dfrac{\sqrt{2}}{5}$
b. $\dfrac{\sqrt{2}}{6}$
c. $\dfrac{\sqrt{2}}{7}$
d. $\dfrac{1}{6}$
e. $\dfrac{1}{7}$

14. The plot at the right is the contour plot of which of these functions?



a. $f(x,y) = \sin(x)\sin(y)$
b. $f(x,y) = x^2 - y^2$
c. $f(x,y) = \sin\left(\sqrt{x^2+y^2}\right)$
d. $f(x,y) = \sin(x) + \sin(y)$
e. $f(x,y) = \sin(xy)$

Work Out: (10 points each. Part credit possible. Show all work.)

15. The pressure P, the temperature T, and the density ρ, of a certain ideal gas are related by $P = 10^{-3}\rho T$. Currently, the temperature is $T = 300$°K and is increasing at 2°K per minute while the density is $\rho = 4\dfrac{\text{gm}}{\text{cm}^3}$ and is decreasing at $0.05\dfrac{\text{gm}}{\text{cm}^3}$ per minute. Consequently, the pressure is currently $P = 10^{-3}\rho T = 10^{-3}(4)(300) = 1.2$ atm. At what rate is P changing and is it increasing or decreasing?

16. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. If the radius and height are measured to be $r = 3\text{cm} \pm 0.02\text{cm}$ and $h = 5\text{cm} \pm 0.03\text{cm}$, then the volume is computed to be $V = \frac{1}{3}\pi 3^2 5 = 15\pi\text{cm}^3$. Use differentials to estimate the error in this computed volume.

17. Find the minimum value of the function $f = x^2 + 2y^2 + 4z^2$ on the plane $x + y + z = 14$.