Name_____

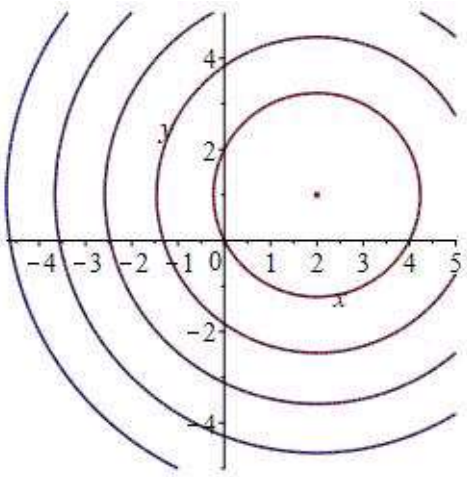
MATH 251 Exam 2 Version H Fall 2018

Sections 200/202 P. Yasskin

1-11	/55	13	/25
12	/20	EC	/5
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. Which of these functions has the contour plot at the right?



a. $\sqrt{x^2+y^2+4x-2y+1}$

b. $\sqrt{x^2+y^2+4x+2y+5}$

c. $\sqrt{x^2+y^2-4x-2y+9}$

d. $x^2+y^2-4x+2y$

e. $x^2+y^2+4x+2y$

2. If $\vec{F} = \langle 2xyz, -3y^2z, 2yz^2 \rangle$, which of the following is FALSE?

a. $\vec{\nabla} \cdot \vec{F} = 0$

b. $\vec{\nabla} \times \vec{F} = \langle 2z^2+3y^2, 2xy, -2xz \rangle$

c. $\vec{F}$ has a vector potential.

d. $\vec{F}$ has a scalar potential.

3. The partial derivative $\left.\frac{\partial f}{\partial x}\right|_{(2,3)}$ gives the

a. slope at $y = 3$ of the x-trace of f with x fixed at 2.

b. slope at $x = 2$ of the x-trace of f with y fixed at 3.

c. slope at $y = 3$ of the y-trace of f with x fixed at 2.

d. slope at $x = 2$ of the y-trace of f with y fixed at 3.

4. Find the tangent plane to the graph of $z = x^2y^3$ at $(x,y) = (2,1)$. The z-intercept is

a. -16

b. 16

c. 4

d. -20

e. 20

5. The equation $x^3z^3 - y^2z^2 = -1$ implicitly defines z as a function of x and y.
Find $\frac{\partial z}{\partial x}$ at $(x,y,z) = (2,3,1)$.

a. 2

b. 1

c. 0

d. -1

e. -2

6. Find the equation of the line perpendicular to the surface $x^3z^3 - y^2z^2 = -1$ at $(x,y,z) = (2,3,1)$.
It intersects the xy-plane at

a. $(0,4,0)$

b. $(-2,5,0)$

c. $(-4,6,0)$

d. $(4,2,0)$

e. $(8,0,0)$

7. The strength, S, of a support beam of length L, width W and height H is given by $S = \frac{WH^2}{L}$. Currently, $L = 50$ cm, $W = 5$ cm and $H = 10$ cm. Use the linear approximation to estimate the change in the strength if L increases by 5 cm, W increases by 0.5 cm and H increases by 2 cm.

a. 2

b. 4

c. 6

d. 8

e. 10

8. Dark Invader is flying through a dark matter field whose density is given by $\delta = xyz^2$. If Dark's current position is $\vec{r}(2) = \langle 3,2,1 \rangle$ and his velocity is $\vec{v}(2) = \langle 1,2,1 \rangle$, find the rate at which the density of dark matter is changing as seen by Dark.

a. 10

b. $10\sqrt{6}$

c. $20\sqrt{6}$

d. 20

e. $\frac{20}{\sqrt{6}}$

9. When there is no wind, a weather balloon floats in the direction of **decreasing** air density. If the air density is $\delta = x^2 + y^2 + z^3$ and the balloon is located at $(x,y,z) = (2,6,1)$, find the vector direction in which the balloon floats.

a. $\left\langle \frac{4}{13}, \frac{12}{13}, \frac{3}{13} \right\rangle$

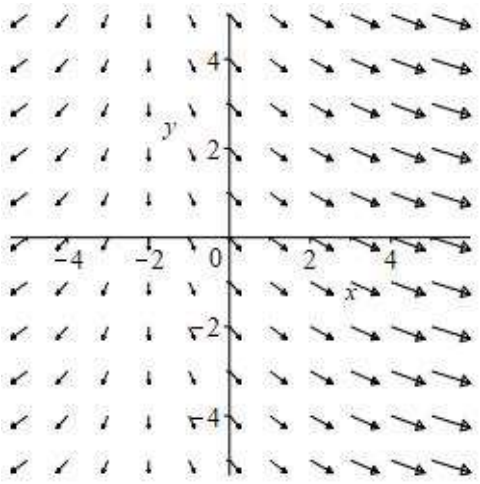
b. $\left\langle \frac{-4}{13}, \frac{12}{13}, \frac{-3}{13} \right\rangle$

c. $\left\langle \frac{-4}{13}, \frac{-12}{13}, \frac{-3}{13} \right\rangle$

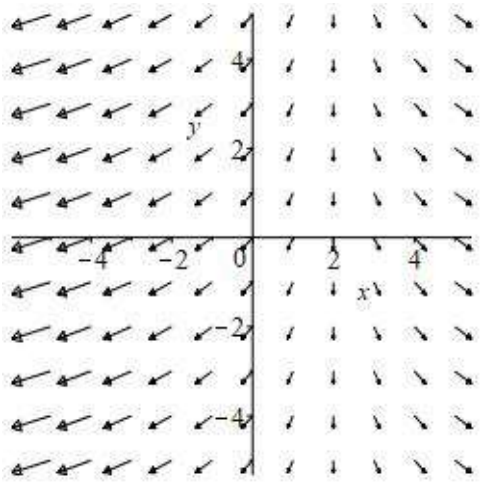
d. $\left\langle \frac{4}{13}, \frac{-12}{13}, \frac{3}{13} \right\rangle$

10. Which is the plot of the vector field $\vec{F} = \langle x - 2, 2 \rangle$?

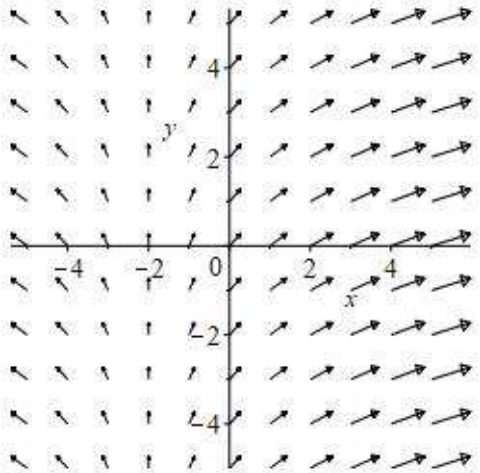
a.



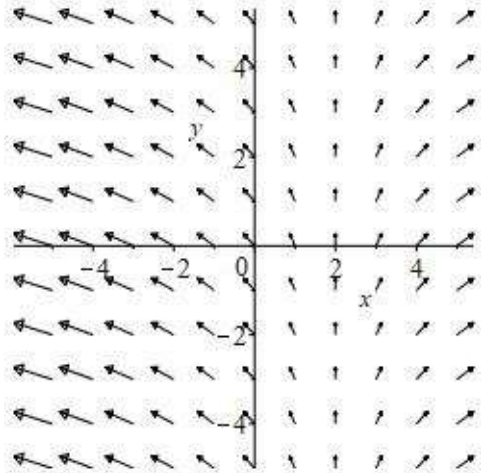
c.



b.



d.



11. Find a scalar potential, $f(x,y,z)$, for $\vec{F} = \left\langle -\frac{yz}{x^2}, \frac{z}{x}, \frac{y}{x} \right\rangle$. Then $f(2,2,2) - f(1,1,1) =$

a. 5

b. 4

c. 3

d. 2

e. 1

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (15 points) Find the point(s), $X = (x,y,z)$, on the hyperboloid $x^2 + y^2 - z^2 = 1$ where the normal vector points in the same direction as $\vec{v} = \langle 1, 5, -5 \rangle$.

13. (15 points+5 points extra credit) Find the point, $X = (x, y, z)$, on the upper half of the hyperboloid $x^2 + y^2 - z^2 = 1$ which is closest to the point $P = (4, 6, 0)$. What is the distance?

You may solve by either method. There is 5 points extra credit for solving by both methods.

Method: **Lagrange Multipliers**::

Method: **Eliminate the Constraint**: