Name________________________

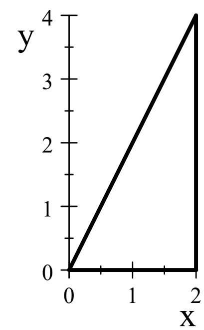
MATH 251 Exam 3 Version H Fall 2018

Sections 504/505 P. Yasskin

1-10	/70	12	/20
11	/20	Total	/110

Multiple Choice: (7 points each. No part credit.)

1. Find the mass of a triangular plate with vertices $(0,0)$, $(2,0)$ and $(2,4)$ if the density is $\delta = x$.



a. $M = 2$

b. $M = 4$

c. $M = 8$

d. $M = \frac{8}{3}$

e. $M = \frac{16}{3}$

2. Find the x-component of the center of mass of a triangular plate with vertices $(0,0)$, $(2,0)$ and $(2,4)$ if the density is $\delta = x$.

a. $\bar{x} = 2$

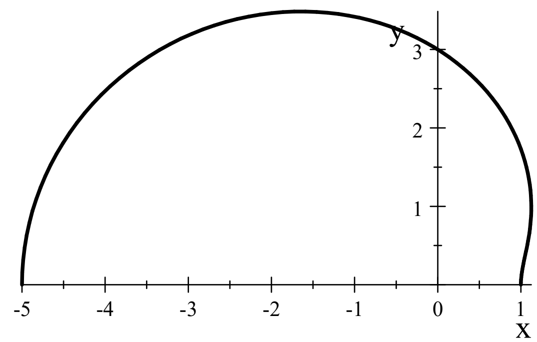
b. $\bar{x} = 4$

c. $\bar{x} = 8$

d. $\bar{x} = \frac{3}{2}$

e. $\bar{x} = \frac{2}{3}$

3. Find the area of the upper half of the limacon $r = 3 - 2\cos\theta$.



a. $A = \frac{9\pi}{2}$

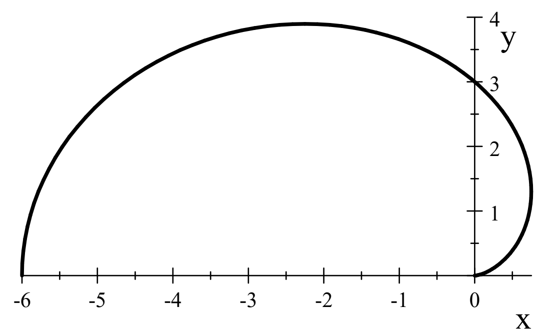
b. $A = \frac{11\pi}{2}$

c. $A = \frac{13\pi}{2}$

d. $A = 9\pi$

e. $A = 11\pi$

4. Given: The area of the upper half of the cardioid $r = 3 - 3\cos\theta$ is $A = \frac{27}{4}\pi$. Find the y-component of its centroid.



a. $\bar{y} = \frac{16}{3\pi}$

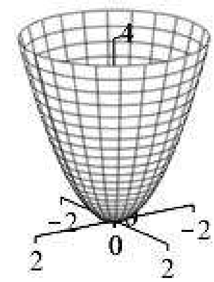
b. $\bar{y} = \frac{3\pi}{16}$

c. $\bar{y} = \frac{16}{9\pi}$

d. $\bar{y} = 36$

e. $\bar{y} = 12$

5. Given: The solid between the paraboloid $z = x^2 + y^2$ and the plane $z = 4$ has volume $V = 8\pi$. Find the z-component of its centroid.



a. $\bar{y} = \dfrac{3}{64\pi}$

b. $\bar{y} = \dfrac{64\pi}{3}$

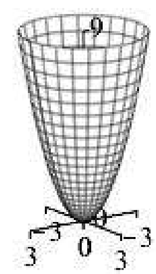
c. $\bar{y} = \dfrac{8}{3}$

d. $\bar{y} = \dfrac{8}{5}$

e. $\bar{y} = \dfrac{16}{5}$

6. Given: The solid between the paraboloid $z = x^2 + y^2$ and the plane $z = 9$ has centroid $(0,0,6)$. If the temperature of the solid is $T = 4 + z$ find the average temperature.

HINT: You don't need to compute any integral.



a. $T_{\text{ave}} = 4$

b. $T_{\text{ave}} = 7$

c. $T_{\text{ave}} = \dfrac{17}{2}$

d. $T_{\text{ave}} = 10$

e. $T_{\text{ave}} = 13$

7. Find the volume of an ice cream cone between the cone $z = \sqrt{x^2 + y^2}$ and the upper piece of the sphere $x^2 + y^2 + z^2 = 9$.

a. $\frac{9}{2}\pi^2$

b. $\frac{9\pi}{\sqrt{2}}$

c. $\frac{18\pi}{\sqrt{2}}$

d. $9\pi\left(1 - \frac{1}{\sqrt{2}}\right)$

e. $18\pi\left(1 - \frac{1}{\sqrt{2}}\right)$

8. Compute $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \frac{1}{(1 + x^2 + y^2)^2}\, dy\, dx$.

HINT: Change coordinates.

a. $\frac{\pi}{40}$

b. $\frac{\pi}{10}$

c. $\frac{\pi}{5}$

d. $\frac{2\pi}{5}$

e. $\frac{-\pi}{40}$

9. Find the work done to push a bead along a wire in the shape of the twisted cubic $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ from $(1,1,1)$ to $(2,4,8)$ if the force is $\vec{F} = \langle z, 2y, x \rangle$.

a. 56

b. 45

c. 30

d. $\frac{45}{2}$

e. 15

10. Find the mass of the conical **surface** $z = \sqrt{x^2+y^2}$ for $z \le 4$ if the surface density is $\delta = z\sqrt{x^2+y^2}$. The surface may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.

SUGGESTION: Do problem 11 first.

a. $M = 128\sqrt{2}\,\pi$

b. $M = 64\sqrt{2}\,\pi$

c. $M = 128\pi$

d. $M = 64\pi$

e. $M = 8\sqrt{2}\,\pi$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (20 points) Find the flux $\iint \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F} = \langle 6xz^2, 6yz^2, z^3 \rangle$ down and out through the conical **surface** $z = \sqrt{x^2 + y^2}$ for $z \le 4$. Follow these steps:

Paramertize the surface as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.

a. Compute the tangent vectors:

$\vec{e}_r = \langle$ ______________, ______________, ______________ $\rangle$

$\vec{e}_\theta = \langle$ ______________, ______________, ______________ $\rangle$

b. Compute the normal vector and check, explain and fix the orientation:

$\vec{N} = \langle$ ______________, ______________, ______________ $\rangle$

c. Evaluate the vector field on the surface:

$\vec{F}\left(\vec{R}(r,\theta)\right) = \langle$ ______________, ______________, ______________ $\rangle$

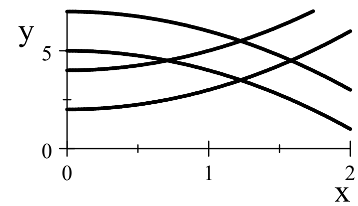
d. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

e. Compute the flux integral:

$\iint \vec{F} \cdot d\vec{S} =$

12. (20 points) Compute the integral $\iint xy\,dA$ over the region in the first quadrant bounded by $y = 2 + x^2$, $y = 4 + x^2$, $y = 5 - x^2$, and $y = 7 - x^2$.



a. Define the curvilinear coordinates u and v by $y = u + x^2$ and $y = v - x^2$.
What are the 4 boundaries in terms of u and v?

$u =$ ________ $u =$ ________ $v =$ ________ $v =$ ________

b. Solve for x and y in terms of u and v. Express the results as a position vector.

$\vec{R}(u,v) = \langle$ ____________________ , ____________________ $\rangle$

c. Find the coordinate tangent vectors:

$\vec{e}_u = \langle$ ____________________ , ____________________ $\rangle$

$\vec{e}_v = \langle$ ____________________ , ____________________ $\rangle$

d. Compute the Jacobian determinant:

$\dfrac{\partial(x,y)}{\partial(u,v)} =$

e. Compute the Jacobian factor:

$J =$

f. Compute the integrand:

$xy =$

g. Compute the integral:

$\iint xy\,dA =$