Name________________________

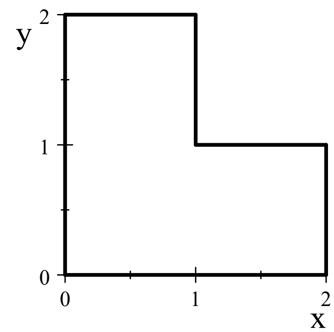
1-10	/60	12	/20
11	/ 5	13	/25
		Total	/110

MATH 251 Final Version A Fall 2018

Sections 505 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. Compute $I = \int_{\partial R} (3y + 2xy^3)\,dx + (8x + 3x^2y^2)\,dy$ over the complete boundary of the L-shape shown at the right traversed counterclockwise.
HINT: Use a theorem.



a. $I = 3$

b. $I = 5$

c. $I = 11$

d. $I = 15$

e. $I = 33$

2. Compute $\int_{(1,1,1)}^{(2,4,8)} \vec{F} \cdot d\vec{s}$ for $\vec{F} = \langle 2xyz, x^2z, x^2y \rangle$ along the curve $\vec{r}(t) = \langle t, t^2t^3 \rangle$.
HINT: Find a scalar potential.

a. 128

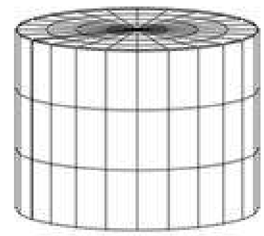
b. 127

c. 0

d. -127

e. -128

3. Compute $\iint_{\partial R} \vec{F} \bullet d\vec{S}$ over the complete surface of the cylinder $x^2 + y^2 \le 4$ for $0 \le z \le 3$ with **outward** orientation, for $\vec{F} = \langle xy^4, x^4 y, 2x^2 y^2 z \rangle$.
HINT: Use a theorem.



a. 64π

b. 48π

c. $\frac{192}{5}\pi$

d. 24π

e. 16π

4. The two legs of a right triangle are $\vec{a}$ and $\vec{b}$ and the hypotenuse is $\vec{c}$. So $\vec{a} \perp \vec{b}$ and $\vec{c} = \vec{a} + \vec{b}$. Given that $\vec{c} = \langle 12, -12, 12 \rangle$ and the direction of $\vec{a}$ is $\hat{a} = \left\langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \right\rangle$, find the magnitude $\vec{b}$.

a. $\left|\vec{b}\right| = 48$

b. $\left|\vec{b}\right| = 24\sqrt{2}$

c. $\left|\vec{b}\right| = 24$

d. $\left|\vec{b}\right| = 12\sqrt{2}$

e. $\left|\vec{b}\right| = 12$

5. An ant is walking across a frying pan where the temperature is $T = x^3 y^2$. If the ant is currently at $P = (2, 3)$ and has velocity $\vec{v} = \langle 2, -4 \rangle$, what is the rate of change of the temperature as seen by the ant?

a. 408

b. 204

c. 24

d. 12

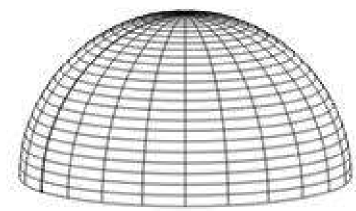
e. 6

6. The point $(1,-2)$ is a critical point of the function $f = \frac{16}{y} - \frac{8}{x} - x^2y^2$.

Classify the point $(1,-2)$ using the Second Derivative Test.

a. Local Minimum
b. Local Maximum
c. Inflection Point
d. Saddle Point
e. Test Fails

7. Find the mass of the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ if the density is $\delta = z$.



a. 2π
b. 4π
c. 8π
d. 16π
e. 32π

8. Find the center of mass of the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ if the density is $\delta = z$.

a. $\left(0, 0, \frac{64\pi}{15}\right)$
b. $\left(0, 0, \frac{32\pi}{15}\right)$
c. $\left(0, 0, \frac{15}{32\pi}\right)$
d. $\left(0, 0, \frac{15}{16}\right)$
e. $\left(0, 0, \frac{16}{15}\right)$

9. Find the equation of the line perpendicular to the hyperboloid $xyz = 6$ at the point $(3,2,1)$.

a. $2x + 3y + 6z = 18$

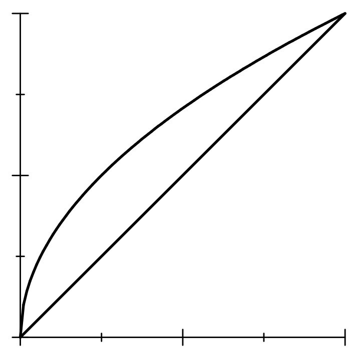
b. $3x + 2y + z = 18$

c. $(x,y,z) = (3 + 2t, 2 - 3t, 1 + 6t)$

d. $(x,y,z) = (2 + 3t, 3 + 2t, 6 + t)$

e. $(x,y,z) = (3 + 2t, 2 + 3t, 1 + 6t)$

10. Find the volume under the surface $z = 2x^2y$ above the region bounded by $y = x$ and $y = 2\sqrt{x}$. The base is shown at the right.



a. $\frac{64}{7}$

b. $\frac{320}{7}$

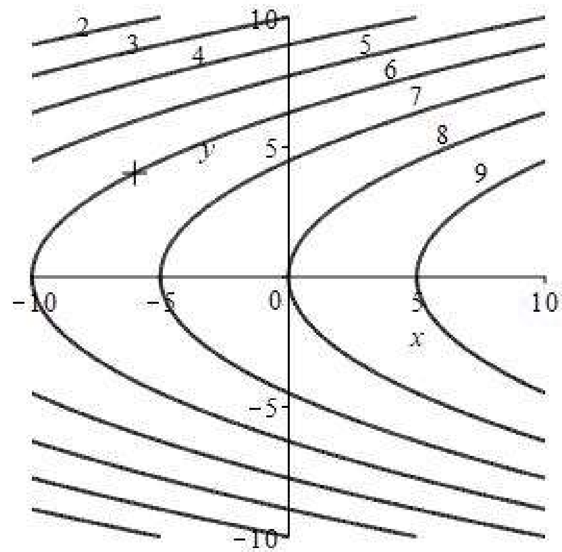
c. $\frac{256}{5}$

d. $\frac{64}{5}$

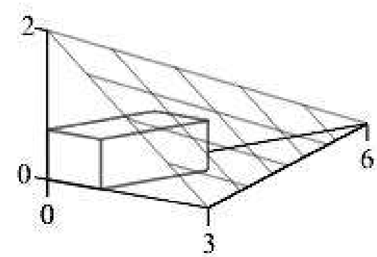
e. $\frac{320}{3}$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (5 points) At the right is the contour plot of a function $f(x,y)$. The contours are labeled by the function values. If you start at the cross at $(-6,4)$ and move so that your velocity is always in the direction of $\vec{\nabla} f$, the gradient of f, roughly sketch your path on the plot.

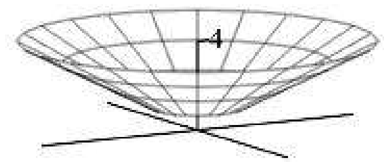


12. (20 points) Find the volume of the largest rectangular box in the first quadrant with three faces in the coordinate planes and one vertex on the plane $2x + y + 3z = 6$.
You do NOT need to check it is a maximum rather than a minimum.



13. (25 points) Verify Stokes' Theorem $\iint\limits_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint\limits_{\partial C} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = \langle 2yz, -2xz, z^2 \rangle$ and the **surface** which is the piece of the cone C given by $z = \frac{1}{2}\sqrt{x^2+y^2}$ between $z = 1$ and $z = 4$ oriented down and out.
Notice that the boundary of C is two circles.
Be sure to check orientations. Use the following steps:



a. Parametrize the cone by $\vec{R}(r,\theta) = \left\langle r\cos\theta, r\sin\theta, \frac{1}{2}r \right\rangle$.
What is the range or r?

______ $\le r \le$ ______

b. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

c. Compute the normal vector and check, explain and fix the orientation:

$\vec{N} =$

d. Compute the curl of $\vec{F}$ and evaluate it on the surface:

$\vec{\nabla} \times \vec{F} =$

$\vec{\nabla} \times \vec{F}\Big|_{\vec{R}(r,\theta)} =$

e. Compute the dot product:

$\vec{\nabla} \times \vec{F} \cdot \vec{N} =$

f. Compute the flux integral:

$\iint_C \vec{F} \cdot d\vec{S} =$

Recall $\vec{F} = \langle 2yz, -2xz, z^2 \rangle$

g. Parametrize the upper circle U:

$\vec{r}(\theta) =$

h. Compute the tangent vector and check, explain and fix the orientation:

$\vec{v}(\theta) =$

i. Evaluate the vector field on the curve:

$\vec{F}\big|_{\vec{r}(\theta)} =$

j. Compute the dot product:

$\vec{F} \cdot \vec{v} =$

k. Compute the integal around U:

$\oint_U \vec{F} \cdot d\vec{s} =$

l. Parametrize the lower circle L:

$\vec{r}(\theta) =$

m. Compute the tangent vector and check, explain and fix the orientation:

$\vec{v}(\theta) =$

n. Evaluate the vector field on the curve:

$\vec{F}\big|_{\vec{r}(\theta)} =$

o. Compute the dot product:

$\vec{F} \cdot \vec{v} =$

p. Compute the integal around L:

$\oint_L \vec{F} \cdot d\vec{s} =$

q. Combine the line integrals:

$\oint_{\partial C} \vec{F} \cdot d\vec{s} =$