Name__________________ ID__________ Section_____

MATH 253 Exam 3 Spring 2003

Sections 501-503 P. Yasskin

1-9	/54	12	/13
10	/13	13	/13
11	/13		

Multiple Choice: (6 points each) Work Out: (13 points each) Extra Credit: (6 points)

1. If $\vec{G} = (x\sin z, y\sin z, xy\cos z)$, then $\vec{\nabla} \cdot \vec{G} =$

a. $(2 - xy)\sin z$
b. $-xy\sin z$
c. $2x\cos z - 2y\cos z$
d. $(\sin z, -\sin z, -xy\sin z)$
e. $(\sin z, \sin z, -xy\sin z)$

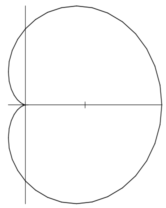
2. If $\vec{G} = (x\sin z, y\sin z, xy\cos z)$, then $\vec{\nabla} \times \vec{G} =$

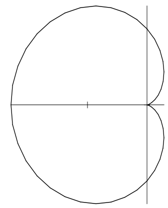
a. $(\sin z, -\sin z, -xy\sin z)$
b. $\big((x-y)\cos z, (y-x)\cos z, 0\big)$
c. $\big((x-y)\cos z, (x-y)\cos z, 0\big)$
d. $2x\cos z - 2y\cos z$
e. 0

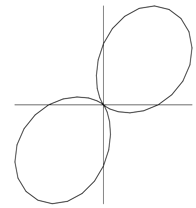
3. If $\vec{F} = \vec{\nabla} f$ and $\vec{\nabla} \times \vec{F} = (12x^2, 12y^2, 12z^2)$ then which of the following could be f?

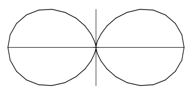
a. $x^4 + y^4 + z^4$
b. $x^4y^4z^4$
c. $x^3y^3 + y^3z^3 + z^3x^3$
d. $x^2y^2 + y^2z^2 + z^2x^2$
e. None of the above

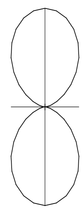
4. Which of the following is the graph of $r = 1 + \cos 2\theta$

a. 

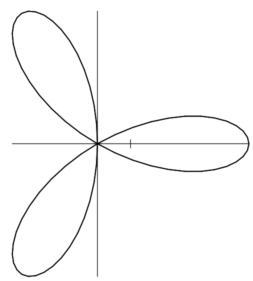
b. 

c. 

d. 

e. 

5. Compute the area inside one leaf of the 3-leaf rose $r = \cos 3\theta$.



a. $\frac{\pi}{12}$
b. $\frac{\pi}{6}$
c. $\frac{\pi}{4}$
d. $\frac{\pi}{3}$
e. $\frac{\pi}{2}$

6. Find the total mass of the hemisphere $0 \le z \le \sqrt{9 - x^2 - y^2}$ if the volume density is $\delta = x^2 + y^2 + z^2$.

a. $\frac{243\pi}{5}$
b. $\frac{486\pi}{5}$
c. 9π
d. $9\pi^2$
e. 18π

7. Find the arc length of the curve $r(t) = \left(2t, t^2, \frac{1}{3}t^3\right)$ between $t = 0$ and $t = 2$.

a. 2
b. $\frac{7}{3}$
c. $\frac{20}{3}$
d. 36
e. 60

8. Compute $\int_{(0,0,0)}^{(4,4,8/3)} (xy + 3z)\, ds$ along the curve $r(t) = \left(2t, t^2, \frac{1}{3}t^3\right)$.

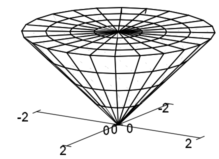
a. 2
b. $\frac{7}{3}$
c. $\frac{20}{3}$
d. 36
e. 56

9. Compute $\int_{(0,0,0)}^{(4,4,8/3)} \vec{F} \cdot d\vec{s}$ along the curve $r(t) = \left(2t, t^2, \frac{1}{3}t^3\right)$ where $\vec{F} = (3z, 2y, x)$.

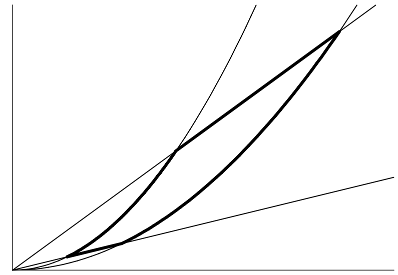
a. 4
b. 8
c. 16
d. 32
e. 64

10. The solid cone $\sqrt{x^2 + y^2} \le z \le 2$ is shown at the right.

Find the mass and center of mass of the cone if its volume density is given by $\delta = x^2 + y^2$.



11. Consider the "diamond shaped" region D bounded by the lines $y = x$ and $y = 3x$ and the parabolas $y = x^2$ and $y = 2x^2$. Compute $\iint_D \frac{y}{x}\,dA$ over the diamond.



Here are some steps to follow:

- Let $u = \frac{y}{x}$ and $v = \frac{y}{x^2}$. Solve for x and y.

- Find the Jacobian factor.

- Express the integrand in terms of u and v.

- Express the boundary curves in terms of u and v.

- Compute $\iint_D \frac{y}{x}\,dA$.

12. Find the total mass of the parametric surface

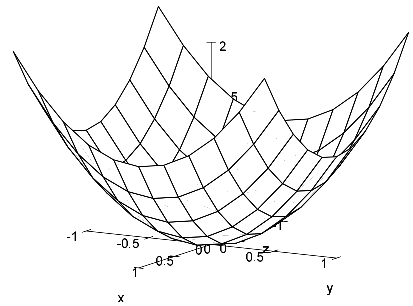
$$\vec{R}(p,q) = (p, q, p^2 + q^2)$$

for $-1 \le p \le 1$ and $-1 \le q \le 1$

if the surface density is

$$\delta = \sqrt{4z+1}\,.$$

HINT: Find $\vec{e}_p$, $\vec{e}_q$, $\vec{N}$ and δ on the surface.



13. Compute $\iint_P \vec{F} \cdot d\vec{S}$ for $\vec{F} = (xz, yz, z^2)$ over the surface of the paraboloid P given by

$$z = x^2 + y^2 \leq 16$$

with normal pointing down and out.

The paraboloid may be parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2).$$

HINT: Find $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$ and $\vec{F}$ on the surface.