Name_____ ID_____ Section_____

	1-10		/50	13		/10
MATH 253	Final Exam	Spring 2003	11	/10	14	/20
Sections 501-503		P. Yasskin	12	/10		

Multiple Choice: (5 points each)   Work Out: (points indicated)

1. Find the angle between the line $\vec{r}(t) = (3 + 2t, -2, 4 - 2t)$ and the normal to the plane $y - z = 4$.

   a. $0°$
   b. $30°$
   c. $45°$
   d. $60°$
   e. $90°$

2. Find the volume of the parallepiped with edges $\vec{A} = (1,2,0)$ $\vec{B} = (2,0,4)$ and $\vec{C} = (0,3,-1)$.

   a. -16
   b. -8
   c. 4
   d. 8
   e. 16

3. Find the equation of the plane tangent to the graph of $f(x,y) = x^3 y + xy^2$ at $(2,1)$. Its z-intercept is

   a. -38
   b. -28
   c. 10
   d. 35
   e. 48

1

4. You are standing at the airport facing North. You look up and see an airplane circling clockwise above the airport. At the moment when the plane is heading due East, in what direction does the plane's binormal point?

   a. Up
   b. Down
   c. North
   d. South
   e. West

5. Suppose $z = \frac{x}{y}$, where $x = x(r,s)$ and $y = y(r,s)$ are functions satisfying

$$x(1,2) = 3 \qquad \frac{\partial x}{\partial r}(1,2) = 5 \qquad \frac{\partial x}{\partial s}(1,2) = 7$$

$$y(1,2) = 4 \qquad \frac{\partial y}{\partial r}(1,2) = 6 \qquad \frac{\partial y}{\partial s}(1,2) = 8$$

Find $\frac{\partial z}{\partial r}$ at $(r,s) = (1,2)$.

   a. $\frac{-1}{16}$
   b. $\frac{1}{8}$
   c. $\frac{1}{4}$
   d. $\frac{3}{4}$
   e. 1

6. Compute $\iint\limits_{R} \frac{1}{x^2 + y^2}\, dx\, dy$ over the ring $9 \le x^2 + y^2 \le 16$.

   a. $2\pi \ln \frac{16}{9}$
   b. $4\pi \ln \frac{16}{9}$
   c. $\pi \ln \frac{4}{3}$
   d. $2\pi \ln \frac{4}{3}$
   e. $4\pi \ln \frac{4}{3}$

7. If $\vec{F} = (xy, yz, zx)$ then $\vec{\nabla} \cdot \vec{F} =$

   a. $y + z + x$
   b. $y - z + x$
   c. $-y + z - x$
   d. $(-y, z, -x)$
   e. $(-y, -z, -x)$

8. Compute $\oint (3x + 4y)\,dx + (2x - 3y)\,dy$ counterclockwise around the edge of the rectangle $1 \le x \le 4,\ \ 2 \le y \le 6$. HINT: Use Green's Theorem.

    a. -36
    b. -24
    c. 12
    d. 24
    e. 36

9. Compute the line integral $\oint y\,dx - x\,dy$ counterclockwise around the semicircle $x^2 + y^2 = 4$ from $(2,0)$ to $(-2,0)$. HINT: Parametrize the circle.

    a. -8π
    b. -4π
    c. π
    d. 4π
    e. 8π

10. Compute $\int \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = \left(\frac{1}{x}, \frac{1}{y} \right)$ along the curve $\vec{r}(t) = \left(e^{\cos(t^2)}, e^{\sin(t^2)} \right)$ for $0 \le t \le \sqrt{\pi}$. Note: $\vec{F} = \vec{\nabla}(\ln x + \ln y)$.

    a. -2
    b. 0
    c. $\dfrac{2}{e}$
   **d.** 1
    e. π

11. (10 points) Find 3 positive numbers x, y and $z,$ whose sum is 120 such that
$f(x,y,z) = xy^2z^3$ is a maximum.

12. (10 points) At the right is the contour
plot of a function $f(x,y)$. If you start
at the dot at $(5,6)$ and move so that
your velocity is always in the direction
of $\vec{\nabla}f,$ the gradient of $f,$ roughly
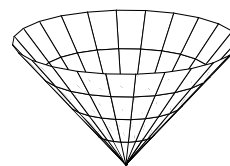sketch your path on the plot.
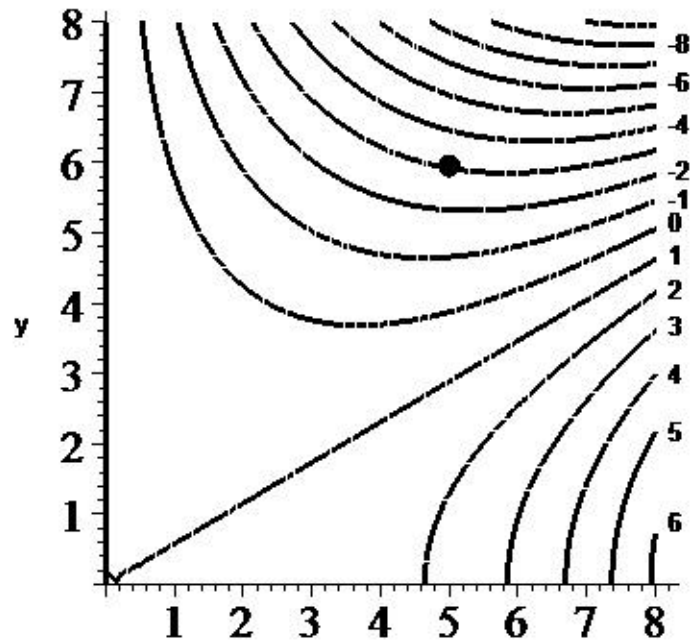
13. (10 points) Find the volume and z-component of the centroid (center of mass) of the solid between the surfaces

$$z = (x^2 + y^2)^{3/2} \qquad \text{and} \qquad z = 8.$$
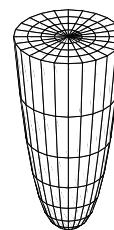
14. (20 points) Use 2 methods to compute $\iint_C \vec{F} \cdot d\vec{S}$ for

$\vec{F} = (5xz, 5yz, z^2)$ over the conical surface C given by

$z = \sqrt{x^2 + y^2} \le 3$ with normal pointing down and out.

a. (7 pts) METHOD 1: Compute $\iint_C \vec{F} \cdot d\vec{S}$ directly as a surface integral using the

parametrization $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.

HINT: Find $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$ and $\vec{F}$ on the cone.

Recall: $\vec{F} = (5xz, 5yz, z^2)$ and C is the conical surface $z = \sqrt{x^2 + y^2} \leq 3$ with normal pointing down and out.

b. (13 pts) METHOD 2:   Compute $\iint_C \vec{F} \cdot d\vec{S}$ by applying Gauss' Theorem

$$\iiint_V \vec{\nabla} \cdot \vec{F} \, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$$ to the solid cone V whose boundary is $\partial V = C + D$

where C is the conical surface and D is the disk at the top of the cone.

HINT: Compute the integrals on V and D and combine them to get the integral on C.