Name_____ ID_____ Section_____

MATH 253   Exam 1   Fall 2003

Sections 504-506   P. Yasskin

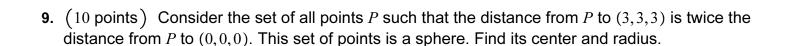| 1-8		/48	11		/15
9		/10	12		/15
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Multiple Choice: (6 points each)   Work Out: (points indicated)

1. A triangle has vertices $A = (0,3,2)$, $B = (-2,3,0)$ and $C = (-2,0,3)$. Find the angle at vertex B.

   a. $\dfrac{\pi}{6}$

   b. $\dfrac{\pi}{3}$

   c. $\dfrac{\pi}{2}$

   d. $\dfrac{2\pi}{3}$

   e. $\dfrac{5\pi}{6}$

2. A triangle has vertices $A = (0,3,2)$, $B = (-2,3,0)$ and $C = (-2,0,3)$. Find the area of the triangle.

   a. 15

   b. 30

   c. $2\sqrt{3}$

   d. $3\sqrt{3}$

   e. $6\sqrt{3}$

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3. If $\vec{u}$ points Up (away from the center of the earth) and $\vec{v}$ points NorthEast, then $\vec{u} \times \vec{v}$ points

    a. Up

    b. Down

    c. SouthEast

    d. SouthWest

    e. NorthWest

4. Find the equation of the plane which is perpendicular to the line $(x,y,z) = (2 - 3t, 3 + t, 1 - t)$ and passes through the point $(-1, 4, 3)$.

    a. $2x + 3y + z = 13$

    b. $2x + 3y + z = -4$

    c. $-3x + y - z = 4$

    d. $-3x + y - z = -4$

    e. $-x + 4y + 3z = 13$

5. Find the arclength of the curve $\vec{r}(t) = \left(\sqrt{2}\, t^2, \sqrt{2}\, t^2, t^3 \right)$ between $t = 0$ and $t = 1$.

    a. $\dfrac{61}{27}$

    b. $\dfrac{122}{9}$

    c. $\dfrac{125}{27}$

    d. 5

    e. $2\ln 2 + \dfrac{3}{4}\sqrt{5} - 2\ln\left(\sqrt{5} + 1\right)$

6. At time $t = 3$ a fly is at the point $\vec{r}(3) = (2,3,1)$ and has velocity $\vec{v}(3) = (1,2,-1)$. If the fly travels in a straight line, where is the fly at time $t = 5$?

   a. $(3,5,0)$

   b. $(0,-1,3)$

   c. $(1,1,2)$

   d. $(4,7,-1)$

   e. $(5,8,1)$

7. At time $t = 3$ a fly is at the point $\vec{r}(3) = (2,3,1)$ and has velocity $\vec{v}(3) = (1,2,-1)$. If the density of fly pheromone is given by $P = xy^2 - 2yz^2$, what is the rate of change of the density of fly pheromone as seen by the fly at $t = 3$?

   a. 41

   b. 36

   c. 17

   d. 14

   e. 1

8. At time $t = 3$ a fly is at the point $\vec{r}(3) = (2,3,1)$ and has velocity $\vec{v}(3) = (1,2,-1)$. If the density of fly pheromone is given by $P = xy^2 - 2yz^2$, in what unit vector direction should the fly fly to increase of the density of fly pheromone as fast as possible?

   a. $\left(\dfrac{1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}} \right)$

   b. $\left(\dfrac{-1}{\sqrt{6}}, \dfrac{-2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right)$

   c. $(-9,-10,12)$

   d. $\left(\dfrac{-9}{5\sqrt{13}}, \dfrac{-2}{\sqrt{13}}, \dfrac{12}{5\sqrt{13}} \right)$

   e. $\left(\dfrac{9}{5\sqrt{13}}, \dfrac{2}{\sqrt{13}}, \dfrac{-12}{5\sqrt{13}} \right)$

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9. (10 points) Consider the set of all points P such that the distance from P to $(3,3,3)$ is twice the distance from P to $(0,0,0)$. This set of points is a sphere. Find its center and radius.

10. (15 points) Circle whether each limit exists or not. If it exists, find the limit using polar coordinates. If it does not exist find two (or more) curves of approach which give different limits.

a. (10) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2}$ .... Exists   Does Not Exist

b. (5) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + x^2 y + y^2}{x^2 + y^2}$ .... Exists   Does Not Exist

11. $\left(15 \text{ points}\right)$ Consider the surface $x^2z^3 + y^2z^3 - 74z = -49$.

    a. (10) Find the tangent plane at the point $(3,4,1)$.

    b. (5) The surface implicitly defines a function $z = f(x,y)$ whose graph (the surface) passes through the point $(3,4,1)$. Use the linear approximation to estimate $f(3.1,3.9)$.

12. $\left(15 \text{ points}\right)$ Given that $z = x^3 + xy^2$ where $x = x(u,v)$ and $y = y(u,v)$ which satisfy

$$x(3,4) = 1 \qquad \left.\frac{\partial x}{\partial u}\right|_{(3,4)} = 5 \qquad \left.\frac{\partial x}{\partial v}\right|_{(3,4)} = 7$$

$$y(3,4) = 2 \qquad \left.\frac{\partial y}{\partial u}\right|_{(3,4)} = 6 \qquad \left.\frac{\partial y}{\partial v}\right|_{(3,4)} = 8$$

Find $\left.\dfrac{\partial z}{\partial u}\right|_{(3,4)}$.