Name________________________ ID____________ Section______

MATH 253 Exam 3 Fall 2003

Sections 504-506 P. Yasskin

1-8	/40	11	/20
9	/10	12	/20
10	/10		/100

Multiple Choice: (5 points each) Work Out: (points indicated)

1. Compute $\iiint x\,dV$ over the solid between the planes $z=0$ and $z=x$, above the triangle with vertices $(0,0)$, $(0,4)$ and $(2,0)$.

a. 4π

b. $\frac{\pi}{2}$

c. $\frac{8}{3}$

d. 4

e. 12

2. For the vector field $\vec{F}=(x^2-y^2z, y^2-z^2x, z^2-x^2y)$, compute $\vec{\nabla}\cdot\vec{\nabla}\times\vec{F}$.

a. $4y-4x$

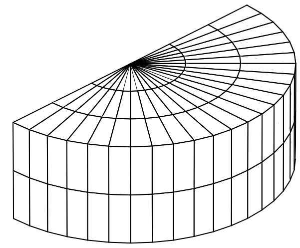
b. $(-2x+2z,-2y+2x,-2z+2y)$

c. $(-2x+2z,2y-2x,-2z+2y)$

d. $64\pi^2$

e. 0

3. Compute the mass of the half cylinder $x^2 + y^2 \le 9$ with $y \ge 0$ for $-1 \le z \le 1$ if the density is $\rho = y$.



a. $\dfrac{3\pi}{2}$

b. $\dfrac{9\pi}{2}$

c. 9π

d. 18π

e. 36

4. Compute the y-component of the center of mass of the half cylinder $x^2 + y^2 \le 9$ with $y \ge 0$ for $-1 \le z \le 1$ if the density is $\rho = y$.

a. 9π

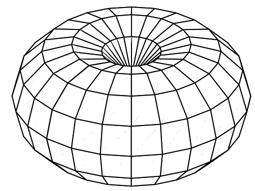
b. $\dfrac{9\pi}{16}$

c. $\dfrac{81\pi}{4}$

d. $\dfrac{9}{8}$

e. $\dfrac{9}{4}$

5. Which of the following integrals will give the volume of the donut given in spherical coordinates by $\rho = \sin\varphi$.



a. $\int_0^{\pi}\int_0^{2\pi}\int_0^{\sin\varphi} \rho^2 \cos\varphi \, d\rho \, d\varphi \, d\theta$

b. $\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \sin\varphi \, d\rho \, d\varphi \, d\theta$

c. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \sin\varphi \, \rho^2 \cos\varphi \, d\rho \, d\varphi \, d\theta$

d. $\int_0^{2\pi}\int_0^{\pi}\int_0^{\sin\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

e. $\int_0^{\pi}\int_0^{2\pi}\int_0^{\sin\varphi} 1 \, d\rho \, d\varphi \, d\theta$

6. For the vector field $\vec{F} = (x^3, y^3, z^3)$, compute $\iiint \vec{\nabla} \cdot \vec{F} \, dV$ over the solid sphere $x^2 + y^2 + z^2 \le 4$.

a. $\frac{384}{5}\pi$

b. $8\pi^2$

c. $16\pi^2$

d. $64\pi^2$

e. 0

7. For the vector field $\vec{F} = (-yz^2, xz^2, z^3)$, compute $\oint \vec{F} \cdot d\vec{s}$ once around the circle $\vec{r}(\theta) = (2\cos\theta, 2\sin\theta, 4)$.

a. 2π

b. 4π

c. 16π

d. 64π

e. 128π

8. Compute $\int_P^Q \vec{F} \cdot d\vec{s}$ along the straight line segment from $P = (1,2,4)$ to $Q = (2,-1,3)$ if $\vec{F} = (yz, xz, xy)$.

a. -14

b. -2

c. 2

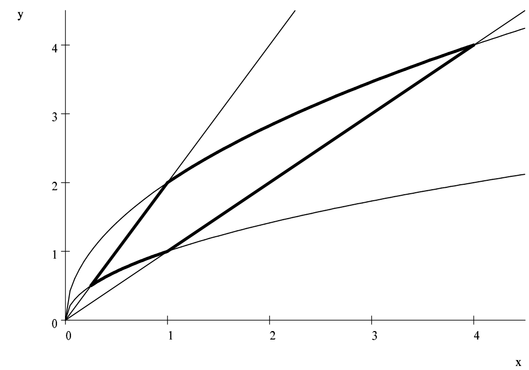
d. 7

e. 14

9. (10 points) Compute $\iint \frac{1}{x^2}\,dx\,dy$ over the diamond shaped region bounded by the curves

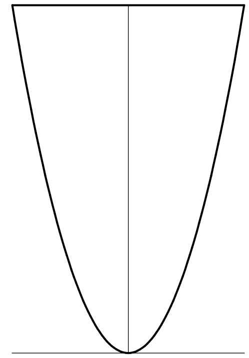
$y = \sqrt{x}$, $y = 2\sqrt{x}$, $y = x$ and $y = 2x$.

HINT: Let $u = \frac{y^2}{x}$ and $v = \frac{y}{x}$.



10. (10 points) Find the area of the parametric surface $\vec{R}(u,v) = (u, v, uv)$ for $u^2 + v^2 \leq 3$.

11. (20 points) Compute $\oint (x^2 y)\,dx + (x^3)\,dy$ counterclockwise around the boundary of the region between the parabola $y = x^2$ and the line $y = 9$ in two ways:



a. Directly by parametrizing the curves.

b. By using Green's theorem: $\iint\limits_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \oint\limits_{\partial R} P\,dx + Q\,dy.$

12. (20 points) For the vector field $\vec{F} = (-yz^2, xz^2, z^3)$, compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the paraboloid $z = x^2 + y^2$ for $z \le 4$ with normal pointing down and out. Use the following steps:
The surface may be parametrized by

$$\vec{R}(r,\theta) = (\quad r\cos\theta \quad , \quad r\sin\theta \quad , \quad r^2 \quad)$$

a. Find the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

b. Find the normal vector:

$\vec{N} =$

c. Compute the curl of $\vec{F}$:

$\vec{\nabla} \times \vec{F} =$

d. Evaluate $\vec{\nabla} \times \vec{F}$ on the surface:

$\vec{\nabla} \times \vec{F}\left(\vec{R}(r,\theta)\right) =$

e. Compute the integral:

$\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$