Name____________________ ID___________ Section_____

MATH 253 Final Exam Fall 2003

Sections 504-506 P. Yasskin

1-10	/50	12	/20
11	/10	13	/20

Multiple Choice: (5 points each) Work Out: (points indicated)

1. Find a parametric equation of the line tangent to the curve $\vec{r}(\theta) = (2\cos\theta, 2\sin\theta, \theta)$ at the point $(-2, 0, \pi)$.

a. $X(t) = (-2t, -2, 1 + \pi t)$

b. $X(t) = (-2, -2t, \pi + t)$

c. $X(t) = (-2t - 2, 0, \pi + t)$

d. $X(t) = (0, -2t - 2, \pi + t)$

e. $X(t) = (-2t - 2, 0, 1 + \pi t)$

2. The density of the fog is given by $\rho = 30 - x^2 - y^2 - z$. If an airplane is at the position $(x, y, z) = \left(2, \sqrt{2}, 4\right)$, in what unit vector direction should the airplane initially travel to get out of the fog as quickly as possible?

a. $\left(\frac{-2}{\sqrt{10}}, \frac{-\sqrt{2}}{\sqrt{10}}, \frac{-2}{\sqrt{10}}\right)$

b. $\left(\frac{2}{\sqrt{10}}, \frac{\sqrt{2}}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right)$

c. $\left(\frac{2}{\sqrt{10}}, \frac{-\sqrt{2}}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right)$

d. $\left(\frac{-4}{5}, \frac{-2\sqrt{2}}{5}, \frac{-1}{5}\right)$

e. $\left(\frac{4}{5}, \frac{2\sqrt{2}}{5}, \frac{1}{5}\right)$

3. Find a parametric equation of the plane tangent to the surface $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$ at the point where $(r,\theta) = (2,\pi)$.

a. $X(s,t) = (-1,0,0) + s(-2,0,\pi) + t(0,-2,1)$

b. $X(s,t) = (1,0,0) + s(2,0,\pi) + t(0,2,1)$

c. $X(s,t) = (0,-2,1) + s(-1,0,0) + t(-2,0,\pi)$

d. $X(s,t) = (-2,0,\pi) + s(-1,0,0) + t(0,-2,1)$

e. $X(s,t) = (2,0,\pi) + s(1,0,0) + t(0,2,1)$

4. Find the non-parametric equation of the plane tangent to the surface $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$ at the point where $(r,\theta) = (2,\pi)$.

a. $y + 2z = 2\pi$

b. $-x = -2y + z$

c. $-x + 2y - z = -2 + \pi$

d. $-x - 2y - z = -2 + \pi$

e. $-y + 2z = 2\pi$

5. Find the non-parametric equation of the plane tangent to the surface $x^2y^2 + x^2z^2 + y^2z^2 = 49$ at the point $(1,2,3)$.

a. $13x + 10y + 5z = 48$

b. $13x - 10y + 5z = 8$

c. $13x + 20y + 15z = 98$

d. $13x - 20y + 15z = 18$

e. $39x + 20y + 5z = 94$

6. Find the non-parametric equation of the plane tangent to the graph of the function $z = x^2y + xy^2$ at the point $(2,1)$.

a. $z = 5x + 8y + 6$

b. $z = -5x - 8y + 6$

c. $z = 5x + 8y - 12$

d. $z = -5x - 8y + 24$

e. $z = 5x + 8y - 6$

7. The point $(1,1)$ is a critical point of the function $f = 4xy - x^4 - y^4$. Using the Second Derivative Test, we find

a. $(1,1)$ is a local maximum.

b. $(1,1)$ is a local minimum.

c. $(1,1)$ is an inflection point.

d. $(1,1)$ is a saddle point.

e. the test fails.

8. Compute $\int_P^Q 2x\,dx + 2y\,dy + 2z\,dz$ along the straight line from $P = (1,2,2)$ to $Q = (3,4,12)$.

a. -10

b. $\sqrt{10}$

c. 10

d. 108

e. 160

9. Compute $\oint 2x\,dx + 2xy\,dy$ counterclockwise around the boundary of the rectangle $1 \le x \le 4$, $2 \le y \le 4$.

a. 6

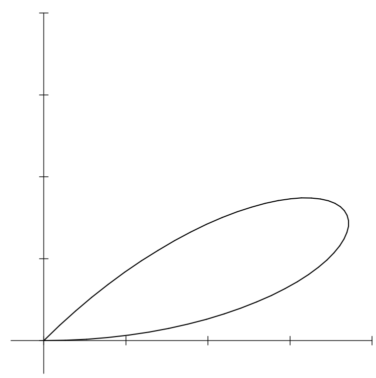
b. 18

c. 24

d. 36

e. 72

10. Find the area of one petal of the 4 leaf rose $r = \sin(4\theta)$.
The petal in the first quadrant.is shown.



a. $\frac{\pi}{16}$

b. $\frac{\pi}{8}$

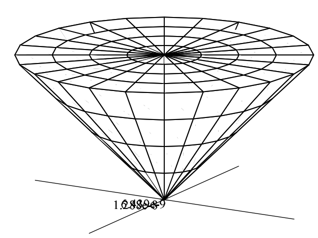
c. $\frac{\pi}{4}$

d. $\frac{\pi}{2}$

e. π

11. (10 points) Find the point in the first octant on the graph of $xy^2z^4 = 32$ which is closest to the origin.
HINTS: What is the square of the distance from a point to the origin? Lagrange multipliers are easier.

12. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (xy^2, yx^2, z^3)$ and the volume above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$.
Use the following steps:

a. Compute the volume integral:

$\vec{\nabla} \cdot \vec{F} =$

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

b. Compute the surface integral over the cone using the parametrization

$\vec{R}(r,\theta) = (\quad r\cos\theta \quad , \quad r\sin\theta \quad , \quad r \quad)$:

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\vec{F}\big(\vec{R}(r,\theta)\big) =$

$\iint\limits_C \vec{F} \bullet d\vec{S} =$

c. Compute the surface integral over the disk using the parametrization

$\vec{R}(r,\theta) = (\quad r\cos\theta \quad , \quad r\sin\theta \quad , \quad 2 \quad)$:

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\vec{F}\big(\vec{R}(r,\theta)\big) =$

$\iint\limits_D \vec{F} \bullet d\vec{S} =$

d. Compute the surface integral over the total boundary:

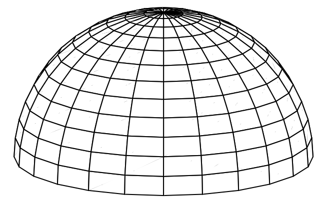
$\iint\limits_{\partial V} \vec{F} \bullet d\vec{S} =$

13. (20 points) Verify Stokes' Theorem $\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial H} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = (y, -x, xz + yz)$

and the hemisphere $z = \sqrt{9 - x^2 - y^2}$.

Use the following steps:



a. Parametrize the boundary curve and compute the line integral:

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial H} \vec{F} \cdot d\vec{s} =$

b. Parametrize the surface and compute the surface integral:

$\vec{R}(\varphi, \theta) =$

$\vec{e}_\varphi =$

$\vec{e}_\theta =$

$\vec{N} =$

$\vec{\nabla} \times \vec{F} =$

$\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2} \qquad \cos^2 A = \frac{1 + \cos(2A)}{2}$$

$$\int_0^{2\pi} \sin^2\theta\, d\theta = \pi \qquad \int_0^{2\pi} \cos^2\theta\, d\theta = \pi$$

$$\int_0^{2\pi} \sin^4\theta\, d\theta = \frac{3}{4}\pi \qquad \int_0^{2\pi} \cos^4\theta\, d\theta = \frac{3}{4}\pi \qquad \int_0^{2\pi} \sin^2\theta\cos^2\theta\, d\theta = \frac{1}{4}\pi$$