MATH 253 Spring 2004 Section 506 P. Yasskin
Maple Quiz Solutions

```
> restart:with(VecCalc):VCalias:
```

#1

```
> f:=[x,y,z] &-> (x^3*y^2*z);
```

$$f := (x, y, z) \to x^3 y^2 z$$

```
> g:=[x,y,z] &-> (x^2+y^2+z^2);
```

$$g := (x, y, z) \to x^2 + y^2 + z^2$$

```
> delf:=Grad(f);
```

$$delf := [(x, y, z) \to 3 x^2 y^2 z, (x, y, z) \to 2 x^3 y z, (x, y, z) \to x^3 y^2]$$

```
> delg:=Grad(g);
```

$$delg := [(x, y, z) \to 2 x, (x, y, z) \to 2 y, (x, y, z) \to 2 z]$$

```
> eqs:=op(equate(delf&@[x,y,z], lambda*delg&@[x,y,z]));
```

$$eqs := 3 x^2 y^2 z = 2 \lambda x, 2 x^3 y z = 2 \lambda y, x^3 y^2 = 2 \lambda z$$

```
> constr:=g(x,y,z)=16;
```

$$constr := x^2 + y^2 + z^2 = 16$$

```
> sol:=solve({eqs,constr}, {x,y,z,lambda});
```

$$sol := \{x = 0, y = \text{RootOf}(\_Z^2 - 16 + z^2), \lambda = 0, z = z\},$$
$$\{x = \text{RootOf}(\_Z^2 - 16 + z^2), y = 0, \lambda = 0, z = z\}, \{$$
$$\lambda = 32\ \text{RootOf}(\_Z^2 - 2, label = \_L2)\ \text{RootOf}(3\ \_Z^2 - 2, label = \_L1),$$
$$y = 4\ \text{RootOf}(3\ \_Z^2 - 1, label = \_L4), z = 2\ \text{RootOf}(3\ \_Z^2 - 2, label = \_L1),$$
$$x = 2\ \text{RootOf}(\_Z^2 - 2, label = \_L2)\}$$

```
> sol1:=allvalues(sol[1]);
```

$$sol1 := \{x = 0, \lambda = 0, z = z, y = \sqrt{16 - z^2}\}, \{x = 0, \lambda = 0, z = z, y = -\sqrt{16 - z^2}\}$$

```
> p1:=subs(sol1[1],[x,y,z]);
```

$$p1 := [0, \sqrt{16 - z^2}, z]$$

```
> p2:=subs(sol1[2],[x,y,z]);
```

$$p2 := [0, -\sqrt{16 - z^2}, z]$$

```
> sol2:=allvalues(sol[2]);
```

$$sol2 := \{y = 0, x = \sqrt{16 - z^2}, \lambda = 0, z = z\}, \{y = 0, x = -\sqrt{16 - z^2}, \lambda = 0, z = z\}$$

```
> p3:=subs(sol2[1],[x,y,z]);
```

$$p3 := [\sqrt{16 - z^2}, 0, z]$$

```
> p4:=subs(sol2[2],[x,y,z]);
```

$$p4 := [-\sqrt{16 - z^2}, 0, z]$$

```
> sol3:=allvalues(sol[3]);
```

$$sol3 := \{y = \frac{4\sqrt{3}}{3}, z = \frac{2\sqrt{6}}{3}, \lambda = \frac{32\sqrt{2}\sqrt{6}}{3}, x = 2\sqrt{2}\},$$

$$\{y=-\frac{4\sqrt{3}}{3}, z=\frac{2\sqrt{6}}{3}, \lambda=\frac{32\sqrt{2}\sqrt{6}}{3}, x=2\sqrt{2}\},$$

$$\{y=\frac{4\sqrt{3}}{3}, x=2\sqrt{2}, z=-\frac{2\sqrt{6}}{3}, \lambda=-\frac{32\sqrt{2}\sqrt{6}}{3}\},$$

$$\{y=-\frac{4\sqrt{3}}{3}, x=2\sqrt{2}, z=-\frac{2\sqrt{6}}{3}, \lambda=-\frac{32\sqrt{2}\sqrt{6}}{3}\},$$

$$\{y=\frac{4\sqrt{3}}{3}, z=\frac{2\sqrt{6}}{3}, x=-2\sqrt{2}, \lambda=-\frac{32\sqrt{2}\sqrt{6}}{3}\},$$

$$\{y=-\frac{4\sqrt{3}}{3}, z=\frac{2\sqrt{6}}{3}, x=-2\sqrt{2}, \lambda=-\frac{32\sqrt{2}\sqrt{6}}{3}\},$$

$$\{y=\frac{4\sqrt{3}}{3}, \lambda=\frac{32\sqrt{2}\sqrt{6}}{3}, x=-2\sqrt{2}, z=-\frac{2\sqrt{6}}{3}\},$$

$$\{y=-\frac{4\sqrt{3}}{3}, \lambda=\frac{32\sqrt{2}\sqrt{6}}{3}, x=-2\sqrt{2}, z=-\frac{2\sqrt{6}}{3}\}$$

```
> p5:=subs(sol3[1],[x,y,z]);
```

$$p5 := \left[2\sqrt{2}, \frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right]$$

```
> p6:=subs(sol3[2],[x,y,z]);
```

$$p6 := \left[2\sqrt{2}, -\frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right]$$

```
> p7:=subs(sol3[3],[x,y,z]);
```

$$p7 := \left[2\sqrt{2}, \frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right]$$

```
> p8:=subs(sol3[4],[x,y,z]);
```

$$p8 := \left[2\sqrt{2}, -\frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right]$$

```
> p9:=subs(sol3[5],[x,y,z]);
```

$$p9 := \left[-2\sqrt{2}, \frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right]$$

```
> p10:=subs(sol3[6],[x,y,z]);
```

$$p10 := \left[-2\sqrt{2}, -\frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right]$$

```
> p11:=subs(sol3[7],[x,y,z]);
```

$$p11 := \left[-2\sqrt{2}, \frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right]$$

```
> p12:=subs(sol3[8],[x,y,z]);
```

$$p12 := \left[-2\sqrt{2}, -\frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right]$$

```
> f &@ p1;
```

$$0$$

```
> f &@ p2;
```

$$0$$

```
> f &@ p3;
```

$$0$$

```
> f &@ p4;
```

$$0$$

```
> f &@ p5;
```

$$\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p6;
```

$$\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p7;
```

$$-\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p8;
```

$$-\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p9;
```

$$-\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p10;
```

$$-\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p11;
```

$$\frac{512\sqrt{2}\sqrt{6}}{9}$$

```
> f &@ p12;
```

$$\frac{512\sqrt{2}\sqrt{6}}{9}$$

So the maximum occurs at

```
> p5;p6;p11;p12;
```

$$\left[2\sqrt{2}, \frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right]$$

$$\left[2\sqrt{2}, -\frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right]$$

$$\left[-2\sqrt{2}, \frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right]$$

$$\left[-2\sqrt{2}, -\frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right]$$

where the function value is

```
> simplify(f &@ p5);
```

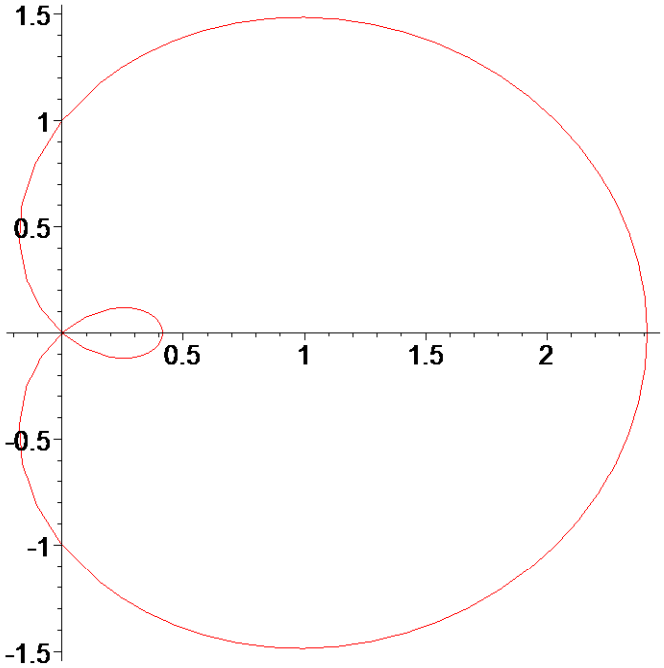
$$\frac{1024\sqrt{3}}{9}$$

#2

```
> r1:=sqrt(2)*cos(theta)+1;
```

$$r1 := \sqrt{2}\cos(\theta) + 1$$

```
> polarplot(r1,theta=0..2*Pi);
```




```
> theta0:=solve(r1=0,theta);
```

$$\theta 0 := \frac{3\pi}{4}$$

```
> x1:=r*cos(theta);
```

$$x1 := r\cos(\theta)$$

```
> y1:=r*sin(theta);
```

$$y1 := r\sin(\theta)$$

```
> rho:=2+y1;
```

$$\rho := 2 + r\sin(\theta)$$

```
> M:=Muint(rho*r, r=0..r1, theta=-theta0..theta0); M:=value(%);
```

$$M := \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \int_{0}^{\sqrt{2}\cos(\theta)+1} (2 + r\sin(\theta))\, r\, dr\, d\theta$$

$$M := 3 + 3\pi$$

```
> xmom:=Muint(x1*rho*r, r=0..r1, theta=-theta0..theta0);
  xmom:=value(%);
```

$$xmom := \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \int_{0}^{\sqrt{2}\cos(\theta)+1} r^2 \cos(\theta)\,(2 + r\sin(\theta))\, dr\, d\theta$$

$$xmom := \frac{7\sqrt{2}}{3} + \frac{9\sqrt{2}\,\pi}{4}$$

```
> xbar:=xmom/M; xbar:=evalf(%);
```

$$xbar := \frac{\frac{7\sqrt{2}}{3} + \frac{9\sqrt{2}\,\pi}{4}}{3 + 3\pi}$$

$$xbar := 1.070145341$$

```
> ymom:=Muint(y1*rho*r, r=0..r1, theta=-theta0..theta0);
  ymom:=value(%);
```

$$ymom := \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \int_{0}^{\sqrt{2}\cos(\theta)+1} r^2 \sin(\theta)\,(2 + r\sin(\theta))\, dr\, d\theta$$

$$ymom := \frac{27\pi}{32} + \frac{53}{60}$$

```
> ybar:=ymom/M; ybar:=evalf(%);
```

$$ybar := \frac{\frac{27\pi}{32} + \frac{53}{60}}{3 + 3\pi}$$

$$ybar := 0.2844358383$$

```
> cm:=[xbar,ybar];
```

$$cm := [1.070145341, 0.2844358383]$$

```
> cmp:=r2p(cm);
```

$$cmp := [1.107300680, 0.2597854139]$$

```
> r2d(cmp[2]);
```

$$14.88460779$$

```
>
```