1-11	/55
12	/12
13	/12
14	/12
15	/12
Total	/103

Name________________________ ID__________________

MATH 253 Exam 1 Spring 2007

Sections 501-503 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the area of the triangle whose vertices are

$P = (3,4,-5)$, $Q = (3,5,-4)$ and $R = (5,2,-5)$.

a. $\sqrt{3}$

b. $2\sqrt{3}$

c. $4\sqrt{3}$

d. 1

e. 6

2. Which of the following is a line perpendicular to the plane $2x - 3y + z = 1$?

a. $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{1}$

b. $\dfrac{x-2}{1} = \dfrac{y-3}{2} = \dfrac{z-1}{3}$

c. $2x + 3y + z = -1$

d. $(x,y,z) = (1+2t, 2+3t, 3+t)$

e. $(x,y,z) = (1+2t, 2-3t, 3+t)$

3. An airplane is travelling due North with constant speed and constant altitude as it flies over College Station. Since its path is part of a circle around the earth, its acceleration points directly toward the center of the earth. In which direction does it binormal $\hat{B}$ point?

a. North

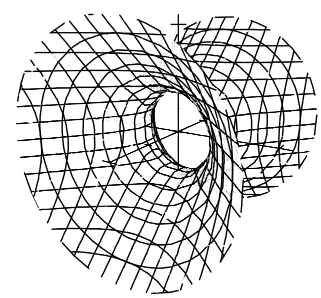
b. East

c. South

d. West

e. Up

4. The plot at the right is which surface?



a. $x^2 - y^2 - z^2 = 4$

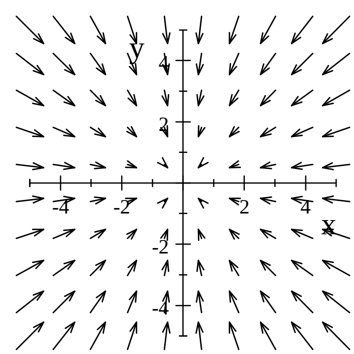
b. $x^2 - y^2 - z^2 = -4$

c. $4x^2 + y^2 + z^2 = 1$

d. $x = 4y^2 - 4z^2$

e. $x = 4y^2 + 4z^2$

5. The plot at the right represents which vector field?



a. $\vec{A} = \langle x, y \rangle$

b. $\vec{B} = \left\langle \dfrac{x}{\sqrt{x^2+y^2}}, \dfrac{y}{\sqrt{x^2+y^2}} \right\rangle$

c. $\vec{C} = \langle -x, -y \rangle$

d. $\vec{D} = \left\langle \dfrac{-x}{\sqrt{x^2+y^2}}, \dfrac{-y}{\sqrt{x^2+y^2}} \right\rangle$

e. $\vec{E} = \langle -y, x \rangle$

6. For the curve $\vec{r}(t) = \left(e^t, \quad \sqrt{2}\, t, \quad e^{-t}\right)$ which of the following is FALSE?

a. $\vec{v} = \left\langle e^t, \quad \sqrt{2}, \quad -e^{-t}\right\rangle$

b. $|\vec{v}| = e^t + e^{-t}$

c. Arc length between $t = 0$ and $t = 1$ is $e + \frac{1}{e}$

d. $\vec{a} = \langle e^t, \quad 0, \quad e^{-t}\rangle$

e. $a_T = e^t - e^{-t}$

7. A wire in the shape of the curve $\vec{r}(t) = \left(e^t, \quad \sqrt{2}\, t, \quad e^{-t}\right)$ has linear mass density $\rho = x + z$. Find its total mass between $t = 0$ and $t = 1$.

a. $\frac{e^2}{2} + 1 - \frac{1}{2e^2}$

b. $\frac{e^2}{2} + 2 - \frac{1}{2e^2}$

c. $\frac{e^2}{2} + 2 + \frac{1}{2e^2}$

d. $e - \frac{1}{e}$

e. $e + \frac{1}{e}$

8. Find the work done to move an object along the curve $\vec{r}(t) = \left(e^t, \quad \sqrt{2}\, t, \quad e^{-t}\right)$ between $t = 0$ and $t = 1$ by the force $\vec{F} = \langle z, 0, -x\rangle$?

a. $2e - \frac{2}{e}$

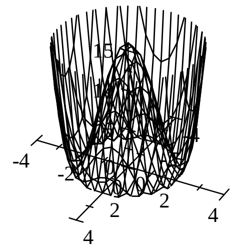
b. $2e + \frac{2}{e}$

c. $e - \frac{1}{e}$

d. $e + \frac{1}{e}$

e. 2

9. The plot at the right is the graph of which function?



a. $f(x,y) = (x^2 + y^2 - 4)^2$

b. $f(x,y) = (x^2 + y^2)^2 - 16$

c. $f(x,y) = x^2 + y^2 - 4$

d. $f(x,y) = (x-2)^2 + (y-2)^2$

e. $f(x,y) = 2x^2 + 2y^2$

10. If $z = x^{3e}e^{3y}$ which of the following is FALSE?

a. $\dfrac{\partial z}{\partial x} = 3ex^{3e-1}e^{3y}$

b. $\dfrac{\partial z}{\partial y} = 3x^{3e}e^{3y}$

c. $\dfrac{\partial^2 z}{\partial x^2} = (9e^2 - 3e)x^{3e-2}e^{3y}$

d. $\dfrac{\partial^2 z}{\partial y \partial x} = 9e^2x^{3e-1}e^{3y}$

e. $\dfrac{\partial^2 z}{\partial x \partial y} = 9ex^{3e-1}e^{3y}$

11. Find the plane tangent to the graph of $z = x\ln(y)$ at the point $(2, e)$. Its z-intercept is

a. $-e$

b. -2

c. 0

d. 2

e. e

Work Out: (12 points each. Part credit possible. Show all work.)

12. Find the vector projection of the vector $\vec{a} = \langle 1, 2, 3 \rangle$ along the vector $\vec{b} = \langle 2, 1, -2 \rangle$.

13. Find the point where the line $\dfrac{x-4}{-1} = \dfrac{y-5}{2} = \dfrac{z-7}{2}$ intersects the plane $x - 3y + z = 6$.

14. The pressure, P, volume, V, and temperature, T, of an ideal gas are related by

$P = \frac{kT}{V}$ for some constant k.

At a certain instant, for a certain sample $k = 5\frac{\text{cm}^3\text{-atm}}{°\text{K}}$, $V = 1000\,\text{cm}^3$, and $T = 300\,°\text{K}$.

At that instant, the volume and temperature are increasing at $\frac{dV}{dt} = 10\frac{\text{cm}^3}{\text{sec}}$, and $\frac{dT}{dt} = 2\frac{°\text{K}}{\text{sec}}$.

At that instant, what is the pressure, is it increasing or decreasing and at what rate?

15. For an adjustable lens, the distance from the lens to the image, v, is related to the distance from the lens to the object, u, and the focal length, f, by the formula

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \qquad \text{or} \qquad v = \frac{fu}{u-f}$$

Currently $f = 4\,\text{cm}$ $u = 6\,\text{cm}$ and so $v = 12\,\text{cm}$

If the focal length is increased by $\Delta f = 0.2\,\text{cm}$, and the distance from the lens to the object is increased by $\Delta u = 0.3\,\text{cm}$, use differentials to estimate how much the image moves.

Does the distance from the lens to the image increase or decrease?