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10	/15	13	/ 6
11	/15		
Total			/110

MATH 253 Final Exam Spring 2007

Sections 501-503 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. Consider the triangle with vertices $A = (1,-1,2)$, $B = (2,3,1)$ and $C = (4,2,2)$. Which vector is perpendicular to the plane of the triangle?

a. $(1,1,3)$

b. $(-1,-1,-3)$

c. $(-1,1,-3)$

d. $(1,1,-3)$

e. $(1,-1,-3)$

2. For the "helix" **curve** $\vec{r}(\theta) = (4\cos\theta, 4\sin\theta, 3\theta)$ find the unit binormal $\hat{B} = \dfrac{\vec{v}\times\vec{a}}{|\vec{v}\times\vec{a}|}$.

a. $(12\sin\theta, 12\cos\theta, 16)$

b. $(-12\sin\theta, -12\cos\theta, -16)$

c. $\left(\frac{3}{5}\sin\theta, \frac{3}{5}\cos\theta, \frac{4}{5}\right)$

d. $\left(-\frac{3}{5}\sin\theta, -\frac{3}{5}\cos\theta, -\frac{4}{5}\right)$

e. $\left(\frac{3}{5}\sin\theta, -\frac{3}{5}\cos\theta, \frac{4}{5}\right)$

3. Find the equation of the plane tangent to $z = xy^2 + x^3y$ at $(x,y) = (1,2)$.
What is the z-intercept?

a. $(0,0,6)$

b. $(0,0,-6)$

c. $(0,0,14)$

d. $(0,0,-14)$

e. $(0,0,26)$

4. The temperature in a room is given by $T = 72 + xyz$. What is the time rate of change of the temperature as seen by a fly located at $P = (3,2,1)$ with velocity $\vec{v} = (2,2,1)$?

a. 4

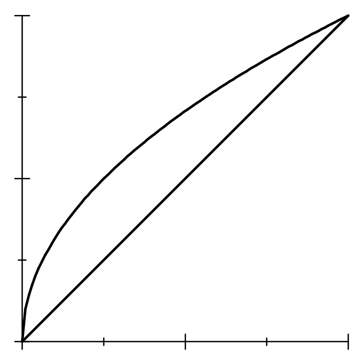
b. 11

c. 16

d. 18

e. 22

5. Find the volume under the surface $z = 2x^2y$ above the region bounded by $y = x$ and $y = 2\sqrt{x}$.
The base is shown at the right.



a. $\frac{256}{5}$

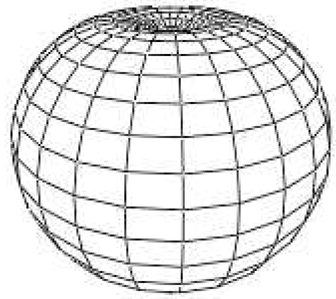
b. $\frac{320}{3}$

c. $\frac{64}{7}$

d. $\frac{320}{7}$

e. $\frac{64}{5}$

6. Find the mass of the solid apple given in spherical coordinates by $\rho = 1 - \cos\phi$ if the volume mass density is $\delta = \rho$.



a. $\frac{2}{5}\pi$

b. $\frac{8}{3}\pi$

c. $\frac{16}{5}\pi$

d. 8π

e. $\frac{64}{15}\pi$

7. Compute $\int_{\vec{r}} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (1 + yz, 1 + xz, 1 + xy)$ along the curve $\vec{r}(t) = (\ln(1+t), t\ln(1+t), t^2\ln(1+t))$ between $t = 0$ and $t = 1$.

HINT: Find a scalar potential and use the Fundamental Theorem of Calculus for Curves.

a. $3\ln 2 + 3(\ln 2)^3$

b. $3\ln 2 + (\ln 2)^3$

c. $3\ln 2 - 3(\ln 2)^3$

d. $3\ln 2 - (\ln 2)^3$

e. $-3\ln 2 + 3(\ln 2)^3$

8. Compute $\oint_C \vec{F} \cdot d\vec{s}$ for $\vec{F} = (x^2y - y^3, x^3 - xy^2)$ counterclockwise around the circle $x^2 + y^2 = 4$.

HINT: Use Green's Theorem.

a. $\frac{16}{3}\pi$

b. 8π

c. $\frac{32}{3}\pi$

d. 16π

e. 32π

9. Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the paraboloid $z = 9 - x^2 - y^2$ for $z \geq 0$,

oriented up, for the vector field $\vec{F} = (z + y, z - x, 2z)$.

HINT: Use Stokes' Theorem. Parametrize the boundary.

a. -18π

b. 0

c. 9π

d. 18π

e. 36π

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (15 points) Find the point (x, y, z) in the first octant on the surface $z = \frac{27}{x} + \frac{64}{y}$ which is closest to the origin.

11. (15 points) Find the mass and z-component of the center of mass of the "twisted cubic" **curve** $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ for $0 \le t \le 1$ if the density is $\rho = 3xz + 3y^2$.

12. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$

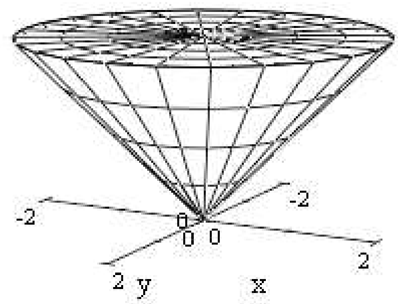
for the vector field $\vec{F} = (xz, yz, -2z^2)$ and

the volume above the cone C: $z = \sqrt{x^2 + y^2}$ for $z \le 2$

and below the disk D: $x^2 + y^2 \le 4$ with $z = 2$.

Be sure to check and explain the orientations.

Use the following steps.



a. Compute the divergence $\vec{\nabla} \cdot \vec{F}$ and the volume integral $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV$.

b. Parametrize the disk, D, and compute the surface integral:
Successively find: $\vec{R}(r,\theta)$, $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, check orientation, $\vec{F}\left(\vec{R}(r,\theta)\right)$, $\iint\limits_D \vec{F} \cdot d\vec{S}$.

c. The cone, C, may be parametrize as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.
Compute the surface integral:
Successively find: $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, check orientation, $\vec{F}\left(\vec{R}(r,\theta)\right)$, $\iint_P \vec{F} \cdot d\vec{S}$

d. Combine $\iint_D \vec{F} \cdot d\vec{S}$ and $\iint_P \vec{F} \cdot d\vec{S}$ to get $\iint_{\partial V} \vec{F} \cdot d\vec{S}$.

13. (6 points) Select the Project that you worked on and then answer the questions in 1 or 2 sentences.

___Gauss' Law and Ampere's Law

One of the electric fields produced a charge density of zero: $\rho_c = \frac{1}{4\pi}\vec{\nabla} \cdot \vec{E} = 0$.

Was there a charge and how did you know? Why was Gauss' Theorem not violated?

___Interpretation of Divergence and Curl

The divergence can be defined using either derivatives or a limit of an integral.

Which Theorem was used to prove their equivalence.

___Skimpy Donut

For the minimal donut, what was the relation between a and b?

For the maximal donut, what was the value of b?

___Volume Between a Surface and Its Tangent Plane

When minimizing over a square, which tangent point (a, b) minimizes the volume?

___Hypervolume of a Hypersphere

The volume enclosed by a sphere of radius R in $\mathbb{R}^n$ is $V_n = k\pi^p R^q$.

What are the values of p and q when $n = 4$? What are the values of p and q when $n = 5$?

___Average Temperatures

What was the shape of the probe used to measure the temperature of the water in the pot?

Which Maple command did you use when Maple was unable to compute the integrals?

___Center of Mass of Planet X

In computing the mass of the water, how did you ensure you only integrated where the land level was below sea level.

Which Maple command did you use when Maple was unable to compute the integrals?