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MATH 253 Quiz 2 Spring 2007

Sections 501-503 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

1. Find the equation of the line through the point $P = (4,3,2)$ in the direction $\vec{v} = (2,1,-1)$. Where does this line pass through the xy-plane?

a. $(8,5,0)$ Correct Choice

b. $(1,2,0)$

c. $(8,-5,0)$

d. $(1,-2,0)$

e. $\left(4, \frac{5}{2}, 0\right)$

The line is: $X = P + t\vec{v}$ or $(x,y,z) = (4+2t, 3+t, 2-t)$

This intersects the xy-plane when $z = 0$ or $2 - t = 0$ or $t = 2$.

So $x = 4 + 2t = 8$ and $y = 3 + t = 5$.

2. Find the equation of the plane through the point $P = (1,3,3)$ with normal $\vec{N} = (4,2,-2)$. Where does this plane pass through the z-axis?

a. $(0,0,-2)$ Correct Choice

b. $(0,0,-1)$

c. $(0,0,1)$

d. $(0,0,2)$

e. $(0,0,4)$

The line is: $\vec{N} \cdot X = \vec{N} \cdot P$ or $4x + 2y - 2z = 4 \cdot 1 + 2 \cdot 3 - 2 \cdot 3 = 4$

This intersects the z-axis when $x = y = 0$. So $-2z = 4$ or $z = -2$.

3. Classify the curve $x^2 - y^2 - 6x - 4y = -4$

a. circle with center $(3,-2)$

b. circle with center $(-3,2)$

c. hyperbola opening left and right Correct Choice

d. hyperbola opening up and down

e. parabola with vertex $(-3,2)$

Complete squares: $(x^2 - 6x + 9) - (y^2 + 4y + 4) = -4 + 9 - 4 = 1$

or $(x-3)^2 - (y+2)^2 = 1$ which is a hyperbola.

Since $(x-3)^2 = 1 + (y+2)^2$ we have $(x-3)^2 \geq 1$.

So the hyperbola opens left and right.

4. Classify the surface $x^2 + y^2 - 4x - 4y - z = -4$

a. elliptic paraboloid opening up Correct Choice
b. elliptic paraboloid opening down
c. hyperboloid of 1 sheet
d. hyperboloid of 2 sheets
e. hyperbolic paraboloid

Complete squares: $(x^2 - 4x + 4) + (y^2 + 4y + 4) - z = -4 + 4 + 4 = 4$
or $z = (x-2)^2 + (y-2)^2 - 4$ which is a paraboloid.
Since the coefficients of x^2 and y^2 are both positive, the paraboloid is elliptic, opening up.

5. Find the point where the line $\frac{x-1}{-1} = \frac{y-5}{2} = z - 7$ intersects the plane $3x - 2y + z = 12$.
HINT: Use the line to write x and y as functions of z.
Solve this on the back of the Scantron. Show all work.

$x - 1 = -1(z - 7)$ $x = -z + 8$ $y - 5 = 2(z - 7)$ $y = 2z - 9$

Plug into the plane:

$3(-z + 8) - 2(2z - 9) + z = 11$ or $-3z + 24 - 4z + 18 + z = 12$ or $-6z = -30$

So $z = 5$ $x = -z + 8 = -5 + 8 = 3$ $y = 2z - 9 = 2 \cdot 5 - 9 = 1$

So the point is: $\boxed{(x,y,z) = (3,1,5)}$

Check: $\frac{x-1}{-1} = \frac{3-1}{-1} = -2$ $\frac{y-5}{2} = \frac{1-5}{2} = -2$ $z - 7 = 5 - 6 = -2$ Good

$3x - 2y + z = 3 \cdot 3 - 2 \cdot 1 + 5 = 12$ Good