Name________________________ ID______________

1-3	/15
4	/10
Total	/25

MATH 253 Quiz 4 Spring 2007
Sections 501-503 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. The point $(1,2)$ is a critical point of $f(x,y) = (2x - x^2)(4y - y^2)$.
Use the Second Derivative Test to classify $(1,2)$ as one of the following:

a. Local Maximum Correct Choice

b. Local Minimum

c. Inflection Point

d. Saddle Point

e. Test Fails

$f_x = (2-2x)(4y-y^2) = 0 \qquad f_y = (2x-x^2)(4-2y) = 0$

$f_{xx} = -2(4y-y^2) \qquad f_{yy} = -2(2x-x^2) \qquad f_{xy} = (2-2x)(4-2y)$

$f_{xx}(1,2) = -8 \qquad f_{yy}(1,2) = -2 \qquad f_{xy}(1,2) = 0 \qquad D = f_{xx}(1,2)f_{yy}(1,2) - f_{xy}(1,2)^2 = 16$

$D > 0$ & $f_{xx} < 0 \quad \Rightarrow$ Local Maximum

2. Find the volume of the solid below the surface $z = 2xy$ above the region between the curves $y = x^2$, $y = 0$ and $x = 2$.

a. $\frac{64}{3}$

b. $\frac{32}{3}$ Correct Choice

c. $\frac{16}{3}$

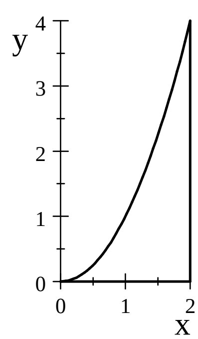
d. $\frac{8}{3}$

e. $\frac{4}{3}$

$$V = \int_0^2 \int_0^{x^2} 2xy\,dy\,dx = \int_0^2 2x\left[\frac{y^2}{2}\right]_{y=0}^{x^2} dx$$

$$= \int_0^2 x(x^4)\,dx = \int_0^2 (x^5)\,dx$$

$$= \left[\frac{x^6}{6}\right]_{x=0}^2 = \left(\frac{2^6}{6}\right) = \frac{32}{3}$$



3. Reverse the order of integration in the integral $\int_0^4 \int_0^{\sqrt{y}} e^{x^3+y^4}\, dx\, dy$

a. $\int_0^{16} \int_0^{x^2} e^{x^4+y^3}\, dy\, dx$

b. $\int_0^{2} \int_{x^2}^{4} e^{x^4+y^3}\, dy\, dx$

c. $\int_0^{16} \int_0^{x^2} e^{x^3+y^4}\, dy\, dx$

d. $\int_0^{2} \int_{x^2}^{4} e^{x^3+y^4}\, dy\, dx$ Correct Choice

e. $\int_0^{2} \int_0^{x^2} e^{x^3+y^4}\, dy\, dx$

4. (10 points) Find the mass and x-component of the center of mass of the plate in the first quadrant bounded by $y = 3 - x$, the x-axis and the y-axis if the surface density is $\rho = y$.

Solve on the back of the Scantron.

This is a triangle.

$$M = \iint \rho\, dA = \int_0^3 \int_0^{3-x} y\, dy\, dx = \int_0^3 \left[\frac{y^2}{2} \right]_{y=0}^{3-x} dx = \int_0^3 \frac{(3-x)^2}{2}\, dx = \frac{1}{2} \int_0^3 (9 - 6x + x^2)\, dx$$

$$= \frac{1}{2} \left[9x - 3x^2 + \frac{x^3}{3} \right]_{x=0}^{3} = \frac{1}{2}(27 - 27 + 9) = \frac{9}{2}$$

$$x\text{-mom} = \iint x\rho\, dA = \int_0^3 \int_0^{3-x} xy\, dy\, dx = \int_0^3 \left[\frac{xy^2}{2} \right]_{y=0}^{3-x} dx = \int_0^3 \frac{x(3-x)^2}{2}\, dx = \frac{1}{2} \int_0^3 (9x - 6x^2 + x^3)\, dx$$

$$= \frac{1}{2} \left[9\frac{x^2}{2} - 2x^3 + \frac{x^4}{4} \right]_{x=0}^{3} = \frac{1}{2} \left(\frac{81}{2} - 54 + \frac{81}{4} \right) = \frac{27}{8}$$

$$\bar{x} = \frac{x\text{-mom}}{M} = \frac{27}{8}\,\frac{2}{9} = \frac{3}{4}$$