Name______________________ ID______________

1-3	/15
4	/10
Total	/25

MATH 253 Quiz 5 Spring 2007

Sections 501-503 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. Find the volume of the solid below the paraboloid $z = 9 - x^2 - y^2$ above the xy-plane.

a. $\frac{3}{2}\pi$

b. 3π

c. $\frac{9}{2}\pi$

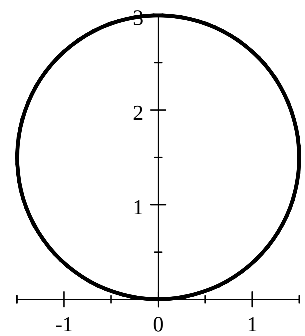
d. $\frac{27}{2}\pi$

e. $\frac{81}{2}\pi$ Correct Choice

$$V = \iint_R (9 - x^2 - y^2)\,dA = \int_0^{2\pi}\int_0^3 (9 - r^2)r\,dr\,d\theta = 2\pi\int_0^3 (9r - r^3)\,dr = 2\pi\left[\frac{9r^2}{2} - \frac{r^4}{4}\right]_{r=0}^{3}$$

$$= 2\pi\left(\frac{81}{2} - \frac{81}{4}\right) = \frac{81}{2}\pi$$

2. Find the center of mass of the circle $r = 3\sin\theta$
if the mass surface density is $\rho = y$.



a. $\left(0, \frac{8}{15}\right)$

b. $\left(0, \frac{15}{8}\right)$ Correct Choice

c. $\left(0, \frac{9}{4}\right)$

d. $\left(0, \frac{4}{9}\right)$

e. $\left(0, \frac{405}{64}\pi\right)$

HINTS: $\int_0^{\pi} \sin^4\theta\,d\theta = \int_0^{\pi} \cos^4\theta\,d\theta = \frac{3}{8}\pi$ $\int_0^{2\pi} \sin^4\theta\,d\theta = \int_0^{2\pi} \cos^4\theta\,d\theta = \frac{3}{4}\pi$

$\int_0^{\pi} \sin^6\theta\,d\theta = \int_0^{\pi} \cos^6\theta\,d\theta = \frac{5}{16}\pi$ $\int_0^{2\pi} \sin^6\theta\,d\theta = \int_0^{2\pi} \cos^6\theta\,d\theta = \frac{5}{8}\pi$

$\rho = y = r\sin\theta$ $\bar{x} = 0$ by symmetry.

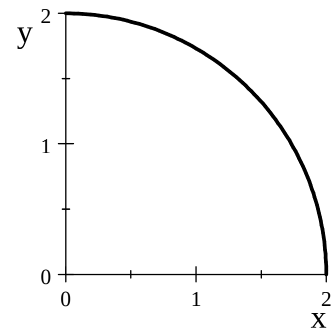
$$M = \iint_R \rho\,dA = \int_0^{\pi}\int_0^{3\sin\theta} (r\sin\theta)r\,dr\,d\theta = \int_0^{\pi}\left[\frac{r^3}{3}\right]_{r=0}^{3\sin\theta}\sin\theta\,d\theta = 9\int_0^{\pi}\sin^4\theta\,d\theta = 9\cdot\frac{3}{8}\pi = \frac{27}{8}\pi$$

$$y\text{-mom} = \iint_R y\rho\,dA = \int_0^{\pi}\int_0^{3\sin\theta} r\sin\theta(r\sin\theta)r\,dr\,d\theta = \int_0^{\pi}\left[\frac{r^4}{4}\right]_{r=0}^{3\sin\theta}\sin^2\theta\,d\theta$$

$$= \frac{81}{4}\int_0^{\pi}\sin^6\theta\,d\theta = \frac{81}{4}\cdot\frac{5}{16}\pi = \frac{405}{64}\pi$$

$$\bar{y} = \frac{y\text{-mom}}{M} = \frac{405\pi}{64}\frac{8}{27\pi} = \frac{15}{8}$$

3. Compute $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\, dy\, dx$



a. $\frac{\pi}{2}(e^4-1)$

b. $\frac{\pi}{2}e^4$

c. $\frac{\pi}{4}(e^4-1)$ Correct Choice

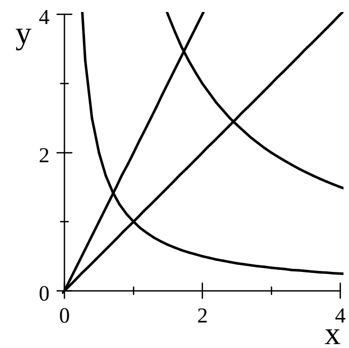
d. $\frac{\pi}{4}e^4$

e. $\frac{\pi}{2}e^3$

$$\int_0^{\pi/2}\int_0^2 e^{r^2} r\, dr\, d\theta = \int_0^{\pi/2}\left[\frac{e^{r^2}}{2}\right]_0^2 d\theta = \int_0^{\pi/2} \frac{e^4-1}{2}\, d\theta = \frac{\pi}{4}(e^4-1)$$

4. Compute $\iint_R y^2 \; dx\, dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{6}{x}, \qquad y = x, \qquad y = 2x$$



FULL CREDIT for integrating in the curvilinear coordinates (u,v) where $u^2 = xy$ and $v^2 = \frac{y}{x}$. (Solve for x and y.)

HALF CREDIT for integrating in rectangular coordinates.

$$\left\{\begin{array}{l} u^2 = xy \\ v^2 = \frac{y}{x} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} u^2v^2 = y^2 \\ \frac{u^2}{v^2} = x^2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} x = \frac{u}{v} \\ y = uv \end{array}\right\}$$

$$J = \left|\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\right| = \left|\left|\begin{array}{cc} \frac{1}{v} & -\frac{u}{v^2} \\ v & u \end{array}\right|\right| = \left|\frac{u}{v} - -\frac{u}{v}\right| = \frac{2u}{v}$$

$xy = 1 \Rightarrow u^2 = 1 \Rightarrow u = 1 \qquad xy = 6 \Rightarrow u^2 = 6 \Rightarrow u = \sqrt{6}$ So: $1 \le u \le \sqrt{6}$

$\frac{y}{x} = 1 \Rightarrow v^2 = 1 \Rightarrow v = 1 \qquad \frac{y}{x} = 2 \Rightarrow v^2 = 2 \Rightarrow v = \sqrt{2}$ So: $1 \le v \le \sqrt{2}$

$$\iint_R y^2 \; dx\, dy = \int_1^{\sqrt{2}}\int_1^{\sqrt{6}} u^2v^2 \; \frac{2u}{v} \; du \; dv = 2\int_1^{\sqrt{2}}\int_1^{\sqrt{6}} u^3 v \; du \; dv$$

$$= 2\left[\frac{u^4}{4}\right]_{u=1}^{\sqrt{6}}\left[\frac{v^2}{2}\right]_{v=1}^{\sqrt{2}} = 2\left[\frac{36}{4} - \frac{1}{4}\right]\left[\frac{2}{2} - \frac{1}{2}\right] = \frac{35}{4}$$