Name______________________________ Sec___________

1-12	/60	14	/15
13	/15	15	/15
Total			/105

MATH 251/253 Exam 2 Spring 2008

Sections 508/200,501,502 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the directional derivative of $f = xyz$ at the point $(x,y,z) = (1,2,3)$ **in the direction of** the vector $\vec{v} = (3,4,12)$.

a. $\dfrac{47}{13}$

b. 30

c. $\dfrac{30}{13}$

d. 54

e. $\dfrac{54}{13}$

2. The point $(2,1)$ is a critical point of the function $f = (x^3 - 3x^2)(y^3 - 3y)$. Use the 2nd Derivative Test to classify $(2,1)$.

a. local minimum

b. local maximum

c. saddle point

d. inflection point

e. The test fails.

3. Compute $\int_1^2 \int_{1/y}^1 ye^{xy}\,dx\,dy$

a. $e^2 - 2e$

b. $e^2 - e$

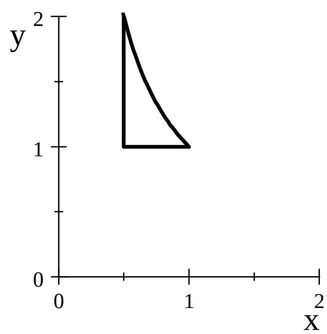
c. $e^2 - 2e - 1$

d. $e^2 - e - 1$

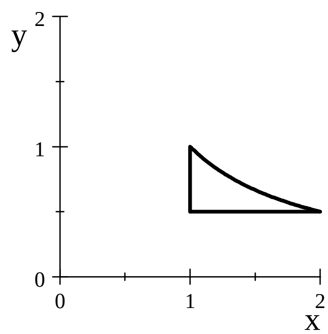
e. $e^2 - 2$

4. The region of integration of the integral in the previous problem is:

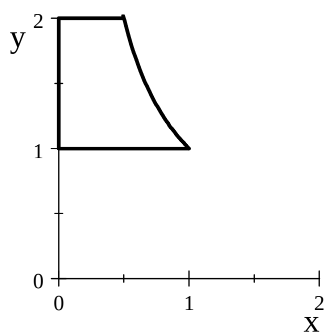
a.



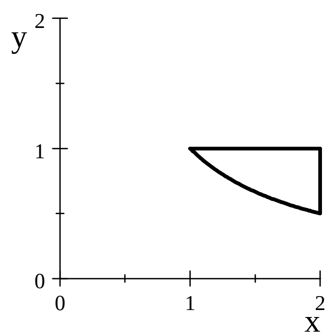
b.



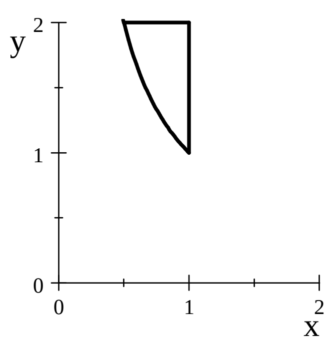
c.



d.



e.



5. Find the mass of the quarter circle $x^2 + y^2 \le 9$ for $x \ge 0$ and $y \ge 0$ if the density is $\rho = \sqrt{x^2 + y^2}$.

a. $\frac{9}{4}\pi$

b. $\frac{9}{2}\pi$

c. $\frac{27}{4}\pi$

d. $\frac{81}{4}\pi$

e. $\frac{243}{2}\pi$

6. Find the center of mass of the quarter circle $x^2 + y^2 \le 9$ for $x \ge 0$ and $y \ge 0$ if the density is $\rho = \sqrt{x^2 + y^2}$.

a. $\left(\frac{4\pi}{9}, \frac{4\pi}{9}\right)$

b. $\left(\frac{2\pi}{9}, \frac{2\pi}{9}\right)$

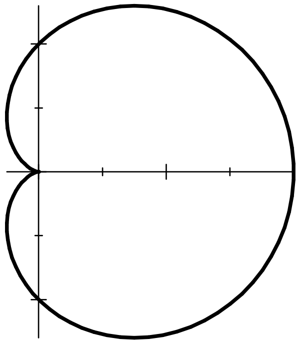
c. $\left(\frac{\pi}{9}, \frac{\pi}{9}\right)$

d. $\left(\frac{9}{2\pi}, \frac{9}{2\pi}\right)$

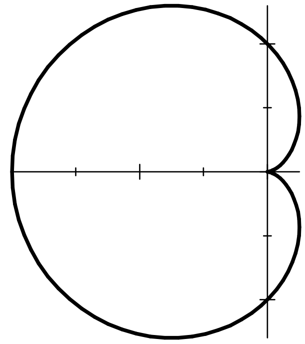
e. $(9,9)$

7. Which of the following is the polar graph of the polar curve $r = 1 - \sin\theta$?

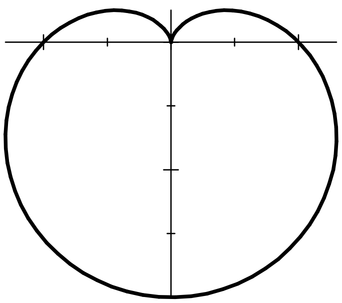
a.



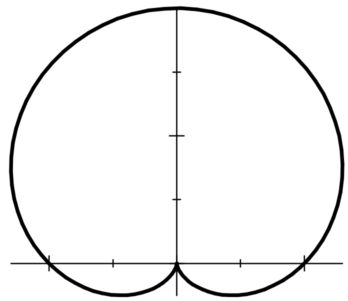
b.



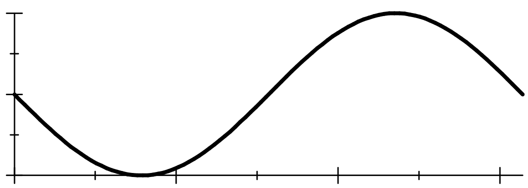
c.



d.



e.



8. The function $f(x,y) = x\sin 2y - y\cos 2x$ satisfies which differential equation?

a. $\vec{\nabla} \cdot \vec{\nabla} f = -4f$

b. $\vec{\nabla} \cdot \vec{\nabla} f = -2f$

c. $\vec{\nabla} \cdot \vec{\nabla} f = 0$

d. $\vec{\nabla} \cdot \vec{\nabla} f = 2f$

e. $\vec{\nabla} \cdot \vec{\nabla} f = 4f$

9. Find the volume above the cone $z = 2\sqrt{x^2 + y^2}$ below the paraboloid $z = 8 - x^2 - y^2$.

a. $\frac{40}{3}\pi$

b. 16π

c. $\frac{56}{3}\pi$

d. $\frac{80}{3}\pi$

e. 32π

10. Find the average value of the function $f(x,y,z) = z^2$ on the hemisphere $x^2 + y^2 + z^2 \le 4$ for $z \ge 0$. HINT: $f_{\text{ave}} = \frac{1}{V}\iiint f\,dV$

a. 0

b. $\frac{4}{5}$

c. $\frac{8}{5}$

d. $\frac{\pi}{4}$

e. $\frac{\pi}{2}$

11. Compute $\int \vec{F} \cdot d\vec{s}$ counterclockwise around the circle $x^2 + y^2 = 4$ with $z = 4$ for the vector field $\vec{F} = (-yz, xz, z^2)$.

a. 2π

b. 4π

c. 8π

d. 16π

e. 32π

12. Compute $\iint \frac{1}{x}\, dS$ on the parametric surface $\vec{R}(u,v) = (u^2 + v^2, u^2 - v^2, 2uv)$ for $1 \le u \le 3$ and $1 \le v \le 4$.

a. $6\sqrt{2}$

b. $12\sqrt{2}$

c. $24\sqrt{2}$

d. $64\sqrt{2}$

e. $272\sqrt{2}$

Work Out: (15 points each. Part credit possible. Show all work.)

13. A plate has the shape of the region between the curves

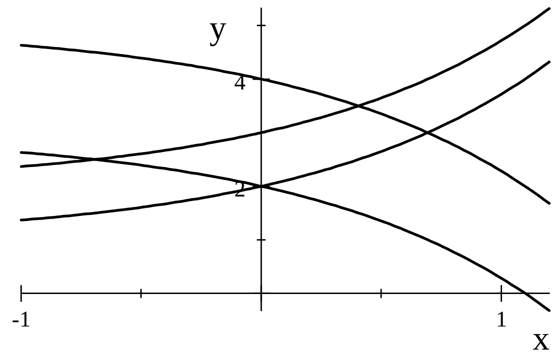
$$y = 1 + \frac{1}{2}e^x, \qquad y = 2 + \frac{1}{2}e^x,$$

$$y = 3 - \frac{1}{2}e^x, \qquad y = 5 - \frac{1}{2}e^x$$

where x and y are measured in centimeters. If the mass density is $\rho = ye^x$ gm/cm^2, find the total mass of the plate.

HINT: Use the curvilinear coordinates

$$u = y - \frac{1}{2}e^x \qquad \text{and} \qquad v = y + \frac{1}{2}e^x$$



14. Find the point in the first octant on the graph of $z = \dfrac{8}{x^2 y}$ closest to the origin.

15. Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the paraboloid $z = x^2 + y^2$ with $z \le 4$ oriented down and out, for the vector field $\vec{F} = (-yz, xz, z^2)$.

HINT: The paraboloid may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.