Name______________________ Sec________

1-13	/65
14	/25
15	/15
Total	/105

MATH 253 Final Exam Spring 2008

Sections 200,501,502 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the equation of the plane containing the two lines:
$\vec{r}_1(s) = (2+3s, -4-2s, 3-s)$ and $\vec{r}_2(t) = (2-t, -4+2t, 3+2t)$

a. $-2x - 5y + 4z = 45$

b. $-2x - 5y + 4z = 1$

c. $-2x - 5y + 4z = -3$

d. $\vec{R}(s,t) = (2+3s-t, -4-2s+2t, 3-s+2t)$

e. $\vec{R}(s,t) = (-2+3s-t, -5-2s+2t, 4-s+2t)$

2. Find the equation of the plane tangent to the graph of the function
$f(x,y) = x^2 + xy + y^2$ at the point $(2,3)$. Then the z-intercept is

a. -38

b. -19

c. 0

d. 19

e. 38

3. Find the arc length of the curve $\vec{r}(t) = (\ln t, 2t, t^2)$ between $(0,2,1)$ and $(1,2e,e^2)$.

Hint: Look for a perfect square.

a. e^2

b. $1+e^2$

c. e^2-1

d. $2+e^2$

e. e^2-2

4. Find the unit binormal $\hat{B}$ of the curve $\vec{r}(t) = (\ln t, 2t, t^2)$ at $t=1$.

Hint: Plug $t=1$ into $\vec{v}$ and $\vec{a}$.

a. $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

b. $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

c. $\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$

d. $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

e. $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

5. The volume of a square pyramid is $V = \frac{1}{3}s^2h$.
If the side of the base s is currently 3 cm and increasing at 2 cm/sec
while the height h is currently 4 cm and decreasing at 1 cm/sec,
is the volume increasing or decreasing and at what rate?

a. increasing at 19 cm$^3/\text{sec}$

b. increasing at 13 cm$^3/\text{sec}$

c. neither increasing nor decreasing

d. decreasing at 13 cm$^3/\text{sec}$

e. decreasing at 19 cm$^3/\text{sec}$

6. Which of the following is a local minimum of $f(x,y) = \sin(x)\cos(y)$?

a. $(0,0)$

b. $\left(\frac{\pi}{2},0\right)$

c. (π,π)

d. $\left(0,\frac{\pi}{2}\right)$

e. None of the above

7. Find the equation of the plane tangent to the surface $x^2z^2 + yz^3 = 11$ at the point $(2,3,1)$. Then the intersection with the x-axis is at

a. $(28,0,0)$

b. $(16,0,0)$

c. $(14,0,0)$

d. $(7,0,0)$

e. $(4,0,0)$

8. Compute $\int \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (y,x)$

along the curve $\vec{r}(t) = (t + \sin t, t + \cos t)$ from $\vec{r}(\pi)$ to $\vec{r}(2\pi)$.

Hint: Find a scalar potential.

a. $3\pi^2 + 3\pi$

b. $3\pi^2 - 3\pi$

c. $3\pi^2 + \pi$

d. $3\pi^2 - \pi$

e. $3\pi - 3\pi^2$

9. Find the mass of the solid hemisphere $x^2 + y^2 + z^2 \leq 4$ for $y \geq 0$ if the density is $\delta = z^2$.

a. $\frac{4}{3}\pi^2$

b. $\frac{8}{3}\pi^2$

c. $\frac{32\pi}{15}$

d. $\frac{64\pi}{15}$

e. $\frac{128\pi}{15}$

10. Find the center of mass of the solid hemisphere $x^2 + y^2 + z^2 \leq 4$ for $y \geq 0$ if the density is $\delta = z^2$.

a. $\left(0, \frac{5}{8}, 0\right)$

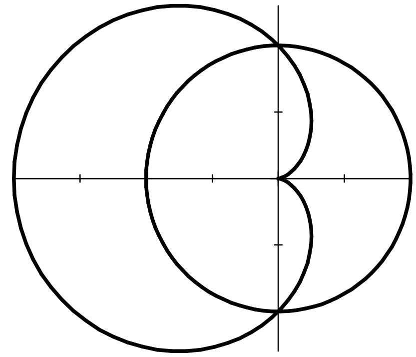
b. $\left(0, \frac{8}{5}, 0\right)$

c. $\left(0, \frac{8\pi}{3}, 0\right)$

d. $\left(0, \frac{3}{8\pi}, 0\right)$

e. $\left(0, \frac{3}{4\pi}, 0\right)$

11. Find the area inside the circle $r = 1$ but outside the cardioid $r = 1 - \cos\theta$.



a. $\frac{\pi}{4}$

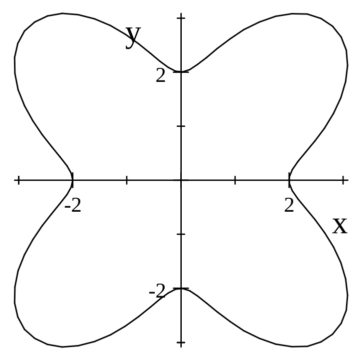
b. $\frac{\pi}{2}$

c. $2 - \frac{\pi}{4}$

d. $2 + \frac{\pi}{4}$

e. $2 - \frac{\pi}{2}$

12. Compute $\oint \vec{\nabla} f \cdot d\vec{s}$ counterclockwise once around the polar curve $r = 3 - \cos(4\theta)$ for the function $f(x,y) = x^2 y$.



a. 2π

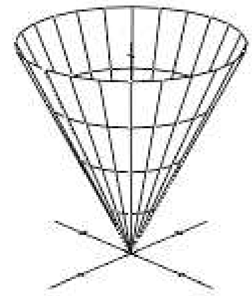
b. 4π

c. 6π

d. 8π

e. 0

13. Stokes' Theorem states $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$



Compute either integral for the cone C given by

$z = 2\sqrt{x^2 + y^2}$ for $z \le 8$ oriented up and in,

and the vector field $\vec{F} = (yz, -xz, z)$.

Note: The cone may be parametrized as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2r)$

The boundary of the cone is the circle $x^2 + y^2 = 16$ with $z = 8$.

a. -768π
b. -256π

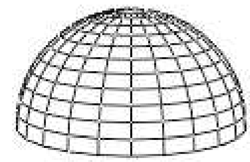
c. 64π

d. 256π

e. 768π

Work Out: (Part credit possible. Show all work.)

14. (25 points) Verify Gauss' Theorem $\iiint_H \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial H} \vec{F} \cdot d\vec{S}$



for the solid hemisphere $x^2 + y^2 + z^2 \le 4$ with $z \ge 0$

and the vector field $\vec{F} = (xz^2, yz^2, x^2 + y^2)$.

Notice that the boundary of the solid hemisphere ∂H consists of the hemisphere surface S given by $x^2 + y^2 + z^2 = 4$ with $z \ge 0$ and the disk D given by $x^2 + y^2 \le 4$ with $z = 0$.

Be sure to check and explain the orientations. Use the following steps:

a. Compute the volume integral by successively finding:

$$\vec{\nabla} \cdot \vec{F}(x,y,z), \quad \vec{\nabla} \cdot \vec{F}(\rho,\theta,\varphi), \quad dV, \quad \iiint_H \vec{\nabla} \cdot \vec{F}\, dV$$

b. Compute the surface integral over the disk by parametrizing the disk and successively finding:

$$\vec{R}(r,\theta), \quad \vec{e}_r, \quad \vec{e}_\theta, \quad \vec{N}, \quad \vec{F}\left(\vec{R}(r,\theta)\right), \quad \iint_D \vec{F} \cdot d\vec{S}$$

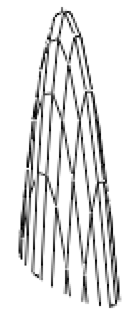
Recall: $\vec{F} = (xz^2, yz^2, x^2 + y^2)$

c. Compute the surface integral over the hemisphere by parametrizing the surface and successively finding:

$$\vec{R}(\theta,\varphi), \quad \vec{e}_\theta, \quad \vec{e}_\varphi, \quad \vec{N}, \quad \vec{F}\left(\vec{R}(\theta,\varphi)\right), \quad \iint_S \vec{F} \bullet d\vec{S}$$

d. Combine $\iint_D \vec{F} \bullet d\vec{S}$ and $\iint_S \vec{F} \bullet d\vec{S}$ to get $\iint_{\partial H} \vec{F} \bullet d\vec{S}$

15. (15 points) A rectangular solid sits on the xy-plane with its top four vertices on the paraboloid $z = 9 - 9x^2 - y^2$. Find the dimensions and volume of the largest such box.



16. (5 points) (Honors only. Replaces #2.) Find the plane tangent to the parametric surface $\vec{R}(u,v) = (u+v, u-v, uv)$ at the point $\vec{R}(1,1) = (2,0,1)$.

Give both the parametric equation and the normal equation of the tangent plane.