Name_____ Sec_____ ID_____

MATH 253	Quiz 1	Spring 2008	Total /25
Sections 200,501,502	Solutions	P. Yasskin	

Multiple Choice: (5 points each)

1. Find the equation of the sphere which passes through the points $(2,1,3)$ and $(4,5,-1)$.

a. $(x-3)^2+(y-3)^2+(z-1)^2=3$
b. $(x-3)^2+(y-3)^2+(z-1)^2=9$ Correct Choice
c. $(x+3)^2+(y+3)^2+(z+1)^2=9$
d. $(x-3)^2+(y-3)^2+(z-1)^2=36$
e. $(x+3)^2+(y+3)^2+(z+1)^2=36$

The center is the midpoint: $\vec{C}=\dfrac{P+Q}{2}=(3,3,1)$

The diameter is the distance: $d(P,Q)=\sqrt{2^2+4^2+4^2}=6$

The radius is half the diameter: $r=3$

The circle is: $(x-3)^2+(y-3)^2+(z-1)^2=9$

2. The angle between the vectors $\vec{u}=(2,-2,4)$ and $\vec{v}=(-1,2,1)$ is

a. acute
b. right
c. obtuse Correct Choice
d. none of these

Since $\vec{u}\cdot\vec{v}=-2-4+4=-2<0,$ the angle is obtuse.

3. A wagon is pulled horizontally from the origin $(0,0)$ to the point $(5,0)$ meters by the force $\vec{F}=(4,3)$ Newtons. Find the work done.

a. 20 Joules Correct Choice
b. 15 Joules
c. 25 Joules
d. $5\sqrt{5}$ Joules
e. $\sqrt{5}$ Joules

The displacement vector is $\vec{D}=(5,0).$ So the work is $W=\vec{F}\cdot\vec{D}=10$ Joules.

4. A triangle has vertices $P = (-1, 2, -3)$, $Q=(3, 2, 1)$, and $R = (-1, -1, 0)$. Find the angle at P.

a. $30°$

b. $45°$

c. $60°$ Correct Choice

d. $90°$

e. $120°$

$\overrightarrow{PQ} = Q - P = (4, 0, 4)$ $\overrightarrow{PR} = R - P = (0, -3, 3)$

$\left|\overrightarrow{PQ}\right| = \sqrt{16+16} = 4\sqrt{2}$ $\left|\overrightarrow{PR}\right| = \sqrt{9+9} = 3\sqrt{2}$ $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 12$

$\cos\theta = \dfrac{12}{4\sqrt{2}\,3\sqrt{2}} = \dfrac{1}{2}$ $\theta = 60°$ (Use a 30-60-90 triangle.)

5. A triangle has vertices $P = (-1, 2, -3)$, $Q=(3, 2, 1)$, and $R = (-1, -1, 0)$. Find the projection of the side $\overrightarrow{PQ}$ onto the base $\overrightarrow{PR}$.

a. $\text{proj}_{\overrightarrow{PR}}\overrightarrow{PQ} = \left(\frac{3}{2}, 0, \frac{3}{2}\right)$

b. $\text{proj}_{\overrightarrow{PR}}\overrightarrow{PQ} = (-1, -1, 0)$

c. $\text{proj}_{\overrightarrow{PR}}\overrightarrow{PQ} = \left(-\frac{2}{3}, -\frac{2}{3}, 0\right)$

d. $\text{proj}_{\overrightarrow{PR}}\overrightarrow{PQ} = (0, -3, 3)$

e. $\text{proj}_{\overrightarrow{PR}}\overrightarrow{PQ} = (0, -2, 2)$ Correct Choice

$\overrightarrow{PQ} = Q - P = (4, 0, 4)$ $\overrightarrow{PR} = R - P = (0, -3, 3)$

$\left|\overrightarrow{PR}\right|^2 = 9 + 9 = 18$ $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 12$

$\text{proj}_{\overrightarrow{PR}}\overrightarrow{PQ} = \dfrac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\left|\overrightarrow{PR}\right|^2}\overrightarrow{PR} = \dfrac{12}{18}(0, -3, 3) = (0, -2, 2)$