Name______________________________ Sec___________

1-14	/56	17	/12
15	/12	18	/12
16	/12		
Total		/104	

MATH 253 Exam 1 Fall 2008

Sections 501-503,200 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. For the triangle with vertices $A = (\sqrt{2}, 2, -2)$, $B = (\sqrt{2}, -1, 1)$ and $C = (3\sqrt{2}, -3, 3)$ find the angle at B.

a. $45°$

b. $60°$

c. $120°$

d. $135°$

e. $150°$

2. For the triangle with vertices $A = (\sqrt{2}, 2, -2)$, $B = (\sqrt{2}, -1, 1)$ and $C = (3\sqrt{2}, -3, 3)$ find the area.

a. 144

b. 72

c. 12

d. $\sqrt{72}$

e. 6

Problems 3 through 8 refer to the curve $\vec{r}(t) = (t^2, 2t, \ln t)$:

3. Find the line tangent to the curve at the point $(1,2,0)$.

a. $(x,y,z) = (2+t, 2+2t, 1)$

b. $(x,y,z) = (2+t, 2-2t, 1)$

c. $(x,y,z) = (2+t, -2-2t, 1)$

d. $(x,y,z) = (1+2t, 2+2t, t)$

e. $(x,y,z) = (1+2t, 2-2t, t)$

4. Find the arc length of the curve between $(1,2,0)$ and $(4,4,\ln 2)$.
HINT: Look for a perfect square.

a. $3+\ln 2$

b. $4+\ln 2$

c. $3+\ln 4$

d. $4+\ln 4$

e. $1+\ln 4$

5. Find the tangential acceleration a_T of the curve.

a. $\dfrac{2t^2+1}{t^2}$

b. $\dfrac{2t^2-1}{t^2}$

c. $\dfrac{4t^4+1}{t^2}$

d. $\dfrac{4t^4-1}{t^2}$

e. $2+\ln t$

Problems 3 through 8 refer to the curve $\vec{r}(t) = (t^2, 2t, \ln t)$:

6. Find the binormal vector $\hat{B}$ of the curve.

a. $\left(\dfrac{-1}{2t^2+1}, \dfrac{-2t}{2t^2+1}, \dfrac{-2t^2}{2t^2+1}\right)$

b. $\left(\dfrac{-1}{2t^2+1}, \dfrac{2t}{2t^2+1}, \dfrac{-2t^2}{2t^2+1}\right)$

c. $\left(\dfrac{-2}{t^2}, \dfrac{-4}{t}, -4\right)$

d. $\left(\dfrac{-2}{t^2}, \dfrac{4}{t}, -4\right)$

e. $(-2, -4t, -4t^2)$

7. Find the mass of a wire in the shape of the curve between $(1,2,0)$ and $(4,4,\ln 2)$, if its linear mass density is $\rho = x + \dfrac{y}{2}e^z$.

a. 8

b. 12

c. 16

d. 18

e. 20

8. Find the work done to move an object along the curve from $(1,2,0)$ to $(4,4,\ln 2)$, under the action of the force $\vec{F} = (y, x, xy)$.

a. $\dfrac{28}{3}$

b. $\dfrac{32}{3}$

c. $\dfrac{56}{3}$

d. $\dfrac{64}{3}$

e. 27

9. Find the plane which passes through the point $P = (2, 4, -1)$ and is perpendicular to the line $(x, y, z) = (1 + 3t, 2 + t, 3 + 2t)$. Its z-intercept is

a. -8

b. -4

c. -2

d. 4

e. 8

10. Find the point where the line $(x, y, z) = (1 - 3t, 2 + t, 1 - 2t)$ intersects the plane $2x + 3y - 3z = -1$. At this point $x + y + z =$

a. 12

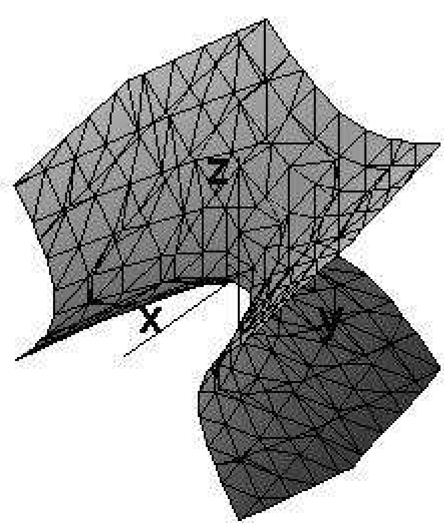
b. 6

c. 5

d. 4

e. -1

11. Which of the following is the equation of the surface?



a. $x^2 - y^2 - z^2 = 0$

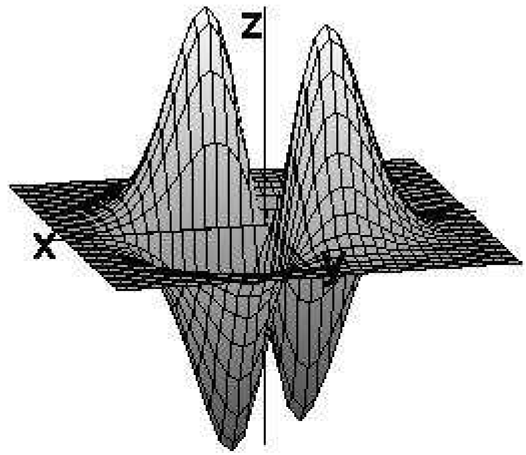
b. $x - y^2 + z^2 = 0$

c. $x + y^2 - z^2 = 0$

d. $x - y^2 - z^2 = 0$

e. $x + y^2 + z^2 = 0$

12. Which of the following is the function graphed?



a. $z = x^2y^2e^{-x^2-y^2}$

b. $z = (x^2 - y^2)e^{-x^2-y^2}$

c. $z = (y^2 - x^2)e^{-x^2-y^2}$

d. $z = xye^{-x^2-y^2}$

e. $z = -xye^{-x^2-y^2}$

13. If $f(x,y) = y\sin(xy)$, which of the following is FALSE?

a. $f_x(1,2) = 4\cos(2)$

b. $f_y(1,2) = \sin(2) + 2\cos(2)$

c. $f_{xx}(1,2) = -4\sin(2) + 4\cos(2)$

d. $f_{xy}(1,2) = 4\cos(2) - 4\sin(2)$

e. $f_{yy}(1,2) = 2\cos(2) - 2\sin(2)$

14. A function $f(x,y)$ satisfies: $f(3,4) = 2$, $f_x(3,4) = -2$, $f_y(3,4) = 3$.
Use the linear approximation to estimate $f(3.2,3.9)$.

a. -0.7

b. 1.3

c. 2.7

d. 5.3

e. 7.3

Work Out: (Points indicated. Part credit possible. Show all work.)

15. (12 points) Duke Skywater is flying across the galaxy in the Millenium Eagle when he find himself passing through a dangerous polaron field. He is currently at the point $\vec{r} = (-1, 1, 2)$ and has velocity $\vec{v} = (0.1, -0.2, 0.2)$ and the polaron density is $\rho = xz^2 + yz^3$.

a. (8 pts) What is the polaron density and its rate of change as currently seen by Duke?

b. (4 pts) In what unit vector direction should Duke travel to **reduce** the polaron density as fast as possible?

16. (12 points) The temperature around a candle is given by $T = 110 - x^2 - y^2 - 2z^2$.

Find the maximum temperature on the plane $4x + 6y + 8z = 42$ and the point where it occur.

17. (12 points) Find an equation of the plane tangent to the graph of the function $f(x,y) = 2x^2y - xy^2$ at $(x,y) = (2,1)$. Then find its z-intercept.

18. (12 points) Find an equation of the plane tangent to the surface $x^4 + x^2y^2 + y^2z^2 = 21$ at the point $(x,y,z) = (1,-2,2)$. Then find its z-intercept.