Name________________________ Sec__________

1-13	/52	15	/15
14	/15	16	/25
Total			/107

MATH 253 Exam 2 Fall 2008

Sections 501-503,200 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. The point $(2,3)$ is a critical point of the function $f = xy + \frac{12}{x} + \frac{18}{y}$.

Apply the Second Derivative Test to classify the point $(2,3)$.

a. local minimum

b. local maximum

c. inflection point

d. saddle point

e. Test Fails

2. Find the volume below the function $z = xy$ above the region in the xy-plane bounded by $x = 4y$ and $x = y^2$.

a. $\frac{80}{3}$

b. $\frac{160}{3}$

c. $\frac{512}{3}$

d. $\frac{640}{3}$

e. $\frac{1024}{3}$

3. Find the mass of the solid below the surface $z = x^2$ above the triangle with vertices $(0,0)$, $(1,0)$ and $(1,2)$, if the density is $\rho = y$.

a. $\frac{1}{5}$

b. $\frac{2}{5}$

c. $\frac{4}{5}$

d. $\frac{6}{5}$

e. $\frac{8}{5}$

4. Find the area of one leaf of the rose $r = \sin(4\theta)$.

a. π

b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{8}$

e. $\frac{\pi}{16}$

5. Find the center of mass of the quarter circle $x^2 + y^2 \leq 9$ in the first quadrant, if the density is $\rho = \sqrt{x^2 + y^2}$.

a. $(\bar{x}, \bar{y}) = \left(\frac{9}{4}, \frac{9}{4}\right)$

b. $(\bar{x}, \bar{y}) = \left(\frac{9}{2}, \frac{9}{2}\right)$

c. $(\bar{x}, \bar{y}) = \left(\frac{2}{9}, \frac{2}{9}\right)$

d. $(\bar{x}, \bar{y}) = \left(\frac{9}{2\pi}, \frac{9}{2\pi}\right)$

e. $(\bar{x}, \bar{y}) = \left(\frac{2\pi}{9}, \frac{2\pi}{9}\right)$

6. Compute $\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{x^2+y^2} e^z \, dz \, dy \, dx$. HINT: Convert to cylindrical coordinates.

a. πe^4

b. $\pi(e^4 - 1)$

c. $\frac{\pi}{2}(e^4 - 3)$

d. $\pi(e^4 - 4)$

e. $\frac{\pi}{2}(e^4 - 5)$

7. Find the average value of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ within the sphere $x^2+y^2+z^2 \le 4$.

a. 3

b. 2

c. $\frac{3}{2}$

d. $\frac{3}{4}$

e. $\frac{3}{8}$

8. Find the mass of a wire in the shape of the curve $\vec{r}(t) = \left(e^t, \sqrt{2}\,t, e^{-t}\right)$ for $0 \le t \le 1$ if the linear density is $\rho = x^2$.

a. $\frac{e^3}{3} + e$

b. $\frac{e^3}{3} + e - \frac{4}{3}$

c. $\frac{1}{2}e^2$

d. $\frac{1}{2}e^2 + \frac{1}{2}$

e. $\frac{1}{2}e^2 - \frac{1}{2}$

9. Compute the line integral $\int_P \vec{\nabla} f \cdot d\vec{s}$ for the function $f = xy$ along the parabola $y = x^2$ from the point $(-1,1)$ to the point $(1,1)$. NOTE: The parabola may be parametrized as $\vec{r}(t) = (t,t^2)$.

a. 0

b. 1

c. 2

d. 3

e. 4

10. Compute the line integral $\oint_C \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (-x^2y, xy^2)$ counterclockwise around the circle $x^2 + y^2 = 9$.

a. 324π

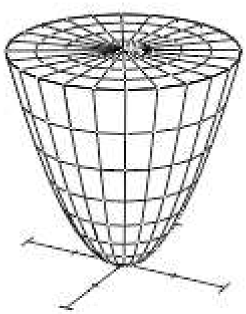
b. 162π

c. 81π

d. $\frac{81\pi}{2}$

e. $\frac{81\pi}{4}$

11. Compute $\iiint_R \vec{\nabla} \cdot \vec{F}\, dV$ for the vector field $\vec{F} = (xz, yz, z^2)$ over the solid bounded by the surfaces $z = x^2 + y^2$ and $z = 4$.



a. $\frac{256\pi}{3}$

b. $\frac{512\pi}{3}$

c. $\frac{256\pi}{5}$

d. $\frac{512\pi}{5}$

e. $\frac{1024\pi}{5}$

12. Find the equation of the plane tangent to the surface $\vec{R}(s,t) = \left(st, \frac{1}{2}s^2 + \frac{1}{2}t^2, \frac{1}{2}s^2 - \frac{1}{2}t^2\right)$ at the point where $(s,t) = (3,1)$.

a. $3x - 5y + 4z = 0$

b. $3x + 5y + 4z = 0$

c. $3x - 5y + 4z = 50$

d. $3x + 5y + 4z = 50$

e. $-6x - 10y - 8z = -50$

13. Find the area of the surface $\vec{R}(s,t) = \left(st, \frac{1}{2}s^2 + \frac{1}{2}t^2, \frac{1}{2}s^2 - \frac{1}{2}t^2\right)$ for $0 \le s \le 1$ and $0 \le t \le 1$.
HINT: Look for a perfect square.

a. $\frac{\sqrt{2}}{3}$

b. $\frac{1}{3}$

c. $\frac{28}{45}\sqrt{2}$

d. $\frac{28}{45}$

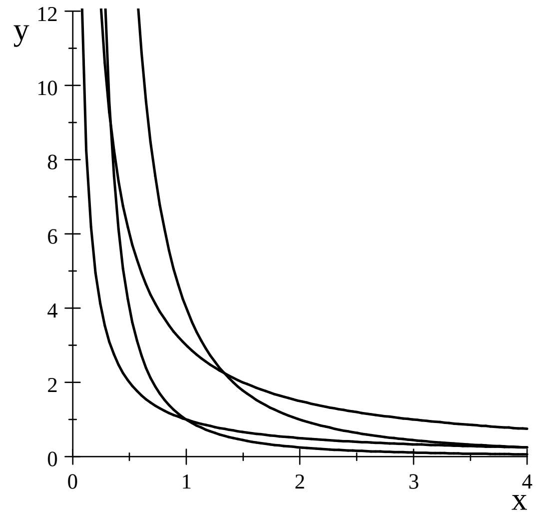
e. $\frac{2\sqrt{2}}{3}$

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (15 points) Compute $\iint x^2 y \, dA$ over the "diamond" shaped region in the first quadrant bounded by the curves

$$y = \frac{1}{x} \qquad y = \frac{3}{x} \qquad y = \frac{1}{x^2} \qquad y = \frac{4}{x^2}$$

HINT: Let $u = xy \qquad v = x^2 y$



15. (15 points) Compute the surface integral $\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (-x^2y, xy^2, z^2)$ over the hemisphere $x^2 + y^2 + z^2 = 9$ with $z \geq 0$ and upward normal, parametrized as $\vec{R}(\theta, \varphi) = (3\sin\varphi\cos\theta, 3\sin\varphi\sin\theta, 3\cos\varphi)$.

16. (25 points) Compute $\iint_S \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (xz, yz, z^2)$ over the complete surface of the solid bounded by the surfaces $z = x^2 + y^2$ and $z = 4$ with outward normal.

(See the figure in #11.)

a. (11 pts) First compute $\iint_P \vec{F} \cdot d\vec{S}$ for the paraboloid $z = x^2 + y^2 \le 4$ which may be parametrized as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.

b. (11 pts) Second compute $\iint_D \vec{F} \cdot d\vec{S}$ for the disk $x^2 + y^2 \le 4$ with $z = 4$ by writing a parametrization.

c. (3 pts) Combine the results from (a) and (b) to obtain $\iint_S \vec{F} \cdot d\vec{S}$.