Name____________________________ Sec__________

1-10	/50	12	/25
11	/15	13	/15
Total			/105

MATH 253 Final Exam Fall 2008

Sections 501-503,200 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find a parametric equation of the line tangent to the curve $\vec{r}(\theta) = (2\sin\theta, 2\cos\theta, \theta)$ at the point $(0,-2,\pi)$.

a. $X(t) = (-2,-2t,1+\pi t)$

b. $X(t) = (0,-2t-2,\pi+t)$

c. $X(t) = (-2t,-2,\pi+t)$

d. $X(t) = (-2t-2,0,\pi+t)$

e. $X(t) = (0,-2t-2,1+\pi t)$

2. The density of the fog is given by $\rho = 30 - x^2 - y^2 - z$. If an airplane is at the position $(x,y,z) = \left(\sqrt{2},2,4\right)$, in what unit vector direction should the airplane initially travel to get out of the fog as quickly as possible?

a. $\left(\frac{-2\sqrt{2}}{5}, \frac{-4}{5}, \frac{-1}{5}\right)$

b. $\left(\frac{2\sqrt{2}}{5}, \frac{4}{5}, \frac{1}{5}\right)$

c. $\left(\frac{-\sqrt{2}}{\sqrt{10}}, \frac{-2}{\sqrt{10}}, \frac{-2}{\sqrt{10}}\right)$

d. $\left(\frac{\sqrt{2}}{\sqrt{10}}, \frac{2}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right)$

e. $\left(\frac{-\sqrt{2}}{\sqrt{10}}, \frac{2}{\sqrt{10}}, \frac{2}{\sqrt{10}}\right)$

3. Find an equation of the plane tangent to the graph of the function $z = x^2 y + xy^2$ at the point $(2,1)$.

a. $z = 5x + 8y + 6$

b. $z = -5x - 8y + 6$

c. $z = -5x - 8y + 24$

d. $z = 5x + 8y - 12$

e. $z = 5x + 8y - 6$

4. Find an equation of the plane tangent to the level surface $x^2y^2 + x^2z^2 + y^2z^2 = 49$ at the point $(1,2,3)$.

a. $13x + 20y + 15z = 98$

b. $13x + 10y + 5z = 48$

c. $13x - 10y + 5z = 8$

d. $13x - 20y + 15z = 18$

e. $39x + 20y + 5z = 94$

5. Find the equation of the plane tangent to the parametric surface $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, \theta)$ at the point where $(r,\theta) = (2,\pi)$.

a. $-x = -2y + z$

b. $-x + 2y - z = -2 + \pi$

c. $-x - 2y - z = -2 + \pi$

d. $-y + 2z = 2\pi$

e. $y + 2z = 2\pi$

6. A satellite is travelling from East to West directly above the equator. In what direction does the binormal $\hat{B}$ point?

a. North

b. South

c. Up

d. Down

e. West

7. Compute $\int_P^Q 2x\,dx + 2y\,dy + 2z\,dz$ along the straight line from $P = (1,-2,2)$ to $Q = (3,-4,12)$.
HINT: Use the Fundamental Theorem of Calculus for Curves.

a. -10

b. $\sqrt{10}$

c. 10

d. 108

e. 160

8. Compute $\oint 2x\,dx + 2xy\,dy$ counterclockwise around the boundary of the rectangle $2 \le x \le 4$, $1 \le y \le 4$.
HINT: Use Green's Theorem.

a. 6

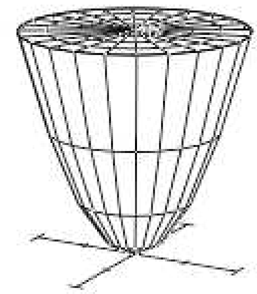
b. 18

c. 30

d. 36

e. 72

9. Compute $\iint_{\partial V} \vec{F} \cdot d\vec{S}$ over the complete boundary of the solid above the paraboloid $z = x^2 + y^2$ and below the plane $z = 4$ with outward normal, for the vector field $\vec{F} = (xy^2, yx^2, z^2)$.
HINT: Use Gauss' Theorem.



a. $-\frac{128}{3}\pi$

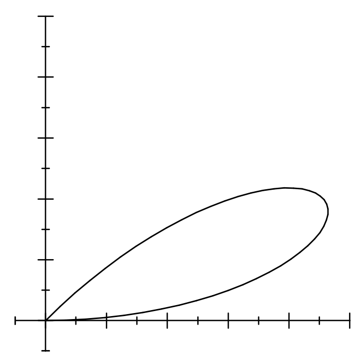
b. 40π

c. 72π

d. $\frac{160}{3}\pi$

e. $\frac{896}{15}\pi$

10. Find the area of one petal of the 4 leaf rose $r = \sin(4\theta)$.
The petal in the first quadrant.is shown.



a. $\frac{\pi}{16}$

b. $\frac{\pi}{8}$

c. $\frac{\pi}{4}$

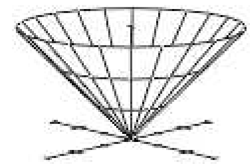
d. $\frac{\pi}{2}$

e. π

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (15 points) Find the point in the first octant on the graph of $4x^4y^2z = 1$ which is closest to the origin. HINTS: What is the square of the distance from a point to the origin? Lagrange multipliers are easier.

12. (25 points) Verify Stokes' Theorem $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$



for the cone C given by $z^2 = x^2 + y^2$ for $z \le 2$

oriented down and out, and the vector field $\vec{F} = (yz^2, -xz^2, z^3)$.

Be sure to check and explain the orientations.

Use the following steps:

a. Note: The cone may be parametrized as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

Compute the surface integral by successively finding:

$\vec{e}_r,\ \ \vec{e}_\theta,\ \ \vec{N},\ \ \vec{\nabla} \times \vec{F},\ \ \vec{\nabla} \times \vec{F}\left(\vec{R}(r,\theta)\right),\ \ \iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

Recall $\vec{F} = (yz^2, -xz^2, z^3)$.

b. Compute the line integral by parametrizing the boundary curve and successively finding:

$$\vec{r}(\theta), \quad \vec{v}, \quad \vec{F}(\vec{r}(\theta)), \quad \oint_{\partial C} \vec{F} \cdot d\vec{s}$$

13. (15 points) Find the mass and center of mass of the $\frac{1}{8}$ of the sphere $x^2 + y^2 + z^2 \le 4$ in the first octant if the density is $\delta = x^2 + y^2 + z^2$.