Name______________________________ Sec__________

MATH 253H Exam 1 Spring 2009

Sections 200 P. Yasskin

1-11	/66	14	/10
12	/10	15	/10
13	/10	Total	/106

Multiple Choice: (6 points each. No part credit.)

1. A triangle has vertices at $P = (9,-6,10)$, $Q = (6,6,6)$ and $R = (10,6,3)$. Find the angle at Q.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

2. Find the tangential acceleration of the curve $\vec{r}(t) = (t^2, 2t, \ln t)$.

a. $a_T = 2 + \frac{1}{t^2}$

b. $a_T = 2 - \frac{1}{t^2}$

c. $a_T = 2t + \frac{1}{t}$

d. $a_T = 2t - \frac{1}{t}$

e. $a_T = t^2 + \ln t$

3. A jet fighter flies directly East at a constant altitude directly above the Equator.
In what direction does the unit binormal vector $\hat{B}$ point?

a. North
b. South
c. West
d. Up
e. Down

4. Find the plane tangent to the graph of the equation $z\ln(y)+x\ln(z)=3e^2$ at the point $(x,y,z)=(e^2,e,e^2)$. Write the equation of the plane in the form $z=Ax+By+C$. What is $A\cdot B\cdot C$?

a. $6e^3$

b. $10e^3$

c. $\frac{3}{4}e^3$

d. $\frac{5}{4}e^3$

e. $4e$

5. Find the plane tangent to the graph of the function $f(x,y)=xy^2-x^3y$ at the point $(x,y)=(1,2)$. Write the equation of the plane in the form $z=Ax+By+C$. What is $A\cdot B\cdot C$?

a. 3

b. 12

c. -2

d. -3

e. -12

6. Consider the function $f(x,y)=3(x+y)^3$. Find the set of **all** points (x,y) where $\vec{\nabla}f=0$.

a. The point $(x,y)=(0,0)$.

b. The point $(x,y)=(1,-1)$.

c. The line $(x,y)=(t,t)$.

d. The line $(x,y)=(t,-t)$.

e. The circle $x^2+y^2=\frac{1}{9}$.

7. The dimensions of a closed rectangular box are measured as 75 cm, 50 cm and 25 cm with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in the calculated surface area of the box.

a. 30 cm^2

b. 60 cm^2

c. 120 cm^2

d. 150 cm^2

e. 1375 cm^2

8. Find the unit vector direction in which the function $f(x,y) = x^3y^2$ **decreases** most rapidly at the point $(x,y) = (2,-1)$.

a. $\left(-\frac{3}{5}, \frac{4}{5}\right)$

b. $\left(\frac{3}{5}, -\frac{4}{5}\right)$

c. $\left(-\frac{4}{5}, \frac{3}{5}\right)$

d. $\left(\frac{4}{5}, -\frac{3}{5}\right)$

e. $\left(-\frac{4}{5}, -\frac{3}{5}\right)$

9. The point $(0,1)$ is a critical point of the function $f(x,y) = x^2y - 3x^2 + 3y^2 - 2y^3$.
Classify the point $(0,1)$ using the Second Derivative Test.

a. Local Mininum

b. Local Maximum

c. Saddle Point

d. Inflection Point

e. Test Fails

10. Suppose $p = p(x,y)$, while $x = x(u,v)$ and $y = y(u,v)$.

Further, you know the following information:

$x(1,2) = 3$	$y(1,2) = 4$	$p(1,2) = 5$	$p(3,4) = 6$
$\frac{\partial p}{\partial x}(1,2) = 7$	$\frac{\partial p}{\partial y}(1,2) = 8$	$\frac{\partial p}{\partial x}(3,4) = 9$	$\frac{\partial p}{\partial y}(3,4) = 10$
$\frac{\partial x}{\partial u}(1,2) = 11$	$\frac{\partial x}{\partial v}(1,2) = 12$	$\frac{\partial x}{\partial u}(3,4) = 13$	$\frac{\partial x}{\partial v}(3,4) = 14$
$\frac{\partial y}{\partial u}(1,2) = 15$	$\frac{\partial y}{\partial v}(1,2) = 16$	$\frac{\partial y}{\partial u}(3,4) = 17$	$\frac{\partial y}{\partial v}(3,4) = 18$

Write out the chain rule for $\frac{\partial p}{\partial v}$. Then use it and the above information to compute $\frac{\partial p}{\partial v}(1,2)$.

a. 134

b. 198

c. 212

d. 249

e. 268

11. A wire has the shape of the helix $\vec{r}(t) = (4\cos t, 4\sin t, 3t)$. It's linear mass density is given by $\rho = x^2 + y^2 + z^2$. Find the total mass of **2 loops** of the wire from $(4,0,0)$ to $(4,0,12\pi)$.

a. $200\pi + 360\pi^3$

b. $320\pi + 960\pi^3$

c. $320\pi + 48\pi^3$

d. $360\pi + 200\pi^3$

e. $960\pi + 320\pi^3$

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (10 points) An object moves around **2 loops** of the helix $\vec{r}(t) = (4\cos t, 4\sin t, 3t)$

from $(4, 0, 0)$ to $(4, 0, 12\pi)$ under the action of a force $\vec{F} = (-y, x, z)$.

Find the work done by the force.

13. (10 points) A cylinder is changing in size.

Currently, the radius is $r = 10$ cm and decreasing at 2 cm/sec.

while the height is $h = 25$ cm and increasing at 4 cm/sec.

Find the current volume. Is it increasing or decreasing and at what rate?

14. (10 points) Find the dimensions and volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the paraboloid $9x^2 + 4y^2 + z = 36$.
You must solve by the Lagrange Multiplier method.

15. (10 points) Determine if each limit exists. If it exists, find it and prove it. If it does not exist, prove it.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - 4y^4}{x^2 + 4y^4}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^2}$