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Name________________________ ID____________ Section______

MATH 253 FINAL EXAM Spring 1998

Sections 501-503 P. Yasskin

Part I: Multiple Choice (5 points each) No Partial Credit

1. $\lim_{n\to\infty} \dfrac{n^2}{n^2+(-1)^n n} =$

a. 0
b. 1
c. 2
d. 4
e. divergent

2. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ is

a. absolutely convergent
b. conditionally convergent
c. divergent
d. none of these

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2+1} =$

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. divergent

4. The series $\sum_{n=1}^{\infty} \frac{n}{n^{1.5}+1}$ is

a. convergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{.5}}$.

b. conv. by the Limit Comp. Test with $\sum_{n=1}^{\infty} \frac{1}{n^{.5}}$ but not by the Comp. Test.

c. divergent by the Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{.5}}$.

d. div. by the Limit Comp. Test with $\sum_{n=1}^{\infty} \frac{1}{n^{.5}}$ but not by the Comp. Test.

e. none of these

5. $\sum_{n=1}^{\infty} \frac{2}{4n^2-1} =$ (Note: $\frac{2}{4n^2-1} = \frac{1}{2n-1} - \frac{1}{2n+1}$)

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. divergent

6. Consider the Taylor series about $x = 0$ for $f(x) = e^{-x}$. What is the minimum degree of the Taylor polynomial you should use to approximate $e^{-0.1}$ to within $\pm 10^{-8}$? Give the degree n of the highest power of x that you need to ***keep***.

a. 1
b. 3
c. 5
d. 7
e. 9

7. Find the volume of the solid under the plane $z = x$ and above the triangle with vertices $(1,1)$, $(2,1)$ and $(1,4)$.

a. 1
b. 2
c. 3
d. 4
e. $\frac{9}{2}$

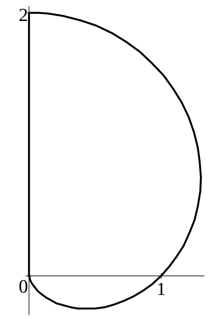
8. A 5 lb mass moves up the helix $\vec{r}(t) = (3\cos t, 3\sin t, 4t)$ for $0 \le t \le \pi$. Find the work done against the force of gravity $\vec{F} = -5\hat{k}$.

a. -4π
b. -5π
c. -20π
d. -80π
e. -100π

9. Compute the line integral $\int_C \vec{F} \bullet d\vec{s}$ counterclockwise around the circle $x^2 + y^2 = 4$ for the vector field $\vec{F} = (-y(x^2+y^2), x(x^2+y^2))$.

a. 2π
b. 4π
c. 8π
d. 16π
e. 32π

10. Find the total mass of a plate bounded by the right half of the cardioid $r = 1 + \sin\theta$ and the y-axis if the mass density is $\rho = 3x$.



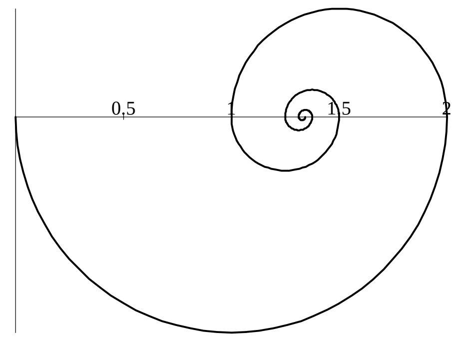
a. 4
b. π
c. 2
d. $\frac{\pi}{2}$
e. $\frac{1}{2}$

11. Find the area of the piece of the paraboloid $z = 9 - x^2 - y^2$ in the first octant.

a. $\frac{\pi}{16}\left[(37)^{3/2} - 1\right]$
b. $\frac{\pi}{4}\left[(37)^{3/2} - 1\right]$
c. $\frac{\pi}{16}(37)^{3/2}$
d. $\frac{\pi}{4}(37)^{3/2}$
e. $\frac{9\pi}{4}$

Part II: Work Out Problems Partial credit will be given.

12. (10 points) The spiral at the right is made from an infinite number of semicircles whose centers are all on the x-axis. The radius of each semicircle is half of the radius of the previous semicircle.



a. Consider the infinite sequence of points where the spiral crosses the x-axis. What is the x-coordinate of the limit of this sequence?

b. What is the total length of the spiral (with an infinite number of semicircles)? Or, is the length infinite?

13. (15 points) Find the interval of convergence for the series $\displaystyle\sum_{n=2}^{\infty} \frac{(x-3)^n}{2^n\, n\, \ln n}$

a. (2 pts) The center of convergence is $c =$ ______.

b. (7 pts) Find the radius of convergence. (Name the test you use.)

$R =$ ______

c. (2 pts) Check the left endpoint. (Name the test you use.)

Circle: { convergent / divergent }

d. (2 pts) Check the right endpoint. (Name the test you use.)

Circle: { convergent / divergent }

e. (2 pts) The interval of convergence is ______.

14. (10 points) Let V be the solid hemisphere $x^2 + y^2 + z^2 \leq 4$ for $z \geq 0$.
Let H be the hemisphere surface $x^2 + y^2 + z^2 = 4$ for $z \geq 0$.
Let D be the disk $x^2 + y^2 \leq 4$ with $z = 0$.

Notice that H and D form the boundaryof V with outward normal provided H is oriented upward and D is oriented downward. Then Gauss' Theorem states

$$\iiint_V \vec{\nabla} \bullet \vec{F}\, dV = \iint_H \vec{F} \bullet d\vec{S} + \iint_D \vec{F} \bullet d\vec{S}$$

Compute $\iint_H \vec{F} \bullet d\vec{S}$ for $\vec{F} = \left(x^3 + y^2 + z^2,\ y^3 + x^2 + z^2,\ z^3 + x^2 + y^2\right)$ using one of the following methods: (Circle the method you choose.)

- Method I: Parametrize H and compute $\iint_H \vec{F} \bullet d\vec{S}$ explicitly.
- Method II: Parametrize D, compute $\iint_D \vec{F} \bullet d\vec{S}$ and $\iiint_V \vec{\nabla} \bullet \vec{F}\, dV$ and solve for $\iint_H \vec{F} \bullet d\vec{S}$.

15. (10 points) Find the point (x, y, z) in the first octant on the surface $z = \frac{27}{x} + \frac{64}{y}$ which is closest to the origin.