

#1

Find the location and value of the minimum of the function $f(x,y,z)=x^2+2y^2+3z^2$ on the plane $x+y+z=11$.

#2

Plot the region between the curves $y=2x^2-2$ and $y=|x|$. If the density is $d=3+y$ compute the mass and y-component of the center of mass of this region.

```
> restart: with(vec_calc): with(linalg): with(student): with(plots):
  vc_aliases:
```

Warning, the protected names norm and trace have been redefined and unprotected

Warning, the name changecoords has been redefined

#1

```
> f:=MF([x,y,z],x^2+2*y^2+3*z^2);
```

$$f := (x, y, z) \rightarrow x^2 + 2y^2 + 3z^2$$

```
> g:=MF([x,y,z],x+y+z);
```

$$g := (x, y, z) \rightarrow x + y + z$$

```
> delf:=GRAD(f);
```

$$delf := [(x, y, z) \rightarrow 2x, (x, y, z) \rightarrow 4y, (x, y, z) \rightarrow 6z]$$

```
> delg:=GRAD(g);
```

$$delg := [1, 1, 1]$$

```
> eqs:=equate(delf(x,y,z),lambda*delg(x,y,z));
```

$$eqs := \{4y = \lambda, 2x = \lambda, 6z = \lambda\}$$

```
> constr:=g(x,y,z)=11;
```

$$constr := x + y + z = 11$$

```
> sol:=solve({op(eqs),constr},{x,y,z,lambda});
```

$$sol := \{x = 6, y = 3, \lambda = 12, z = 2\}$$

```
> pt:=subs(sol,[x,y,z]);
```

$$pt := [6, 3, 2]$$

```
> f(op(pt));
```

66

```
#2
```

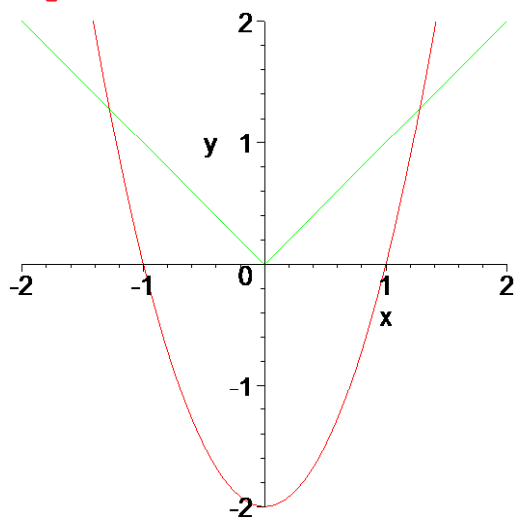
```
> f:=2*x^2-2;
```

$$f := 2x^2 - 2$$

```
> g:=abs(x);
```

$$g := |x|$$

```
> plot([f,g], x=-2..2, y=-2..2);
```



```
> sol:=solve(f=g, x);
```

$$\text{sol} := \frac{1}{4} + \frac{\sqrt{17}}{4}, -\frac{1}{4} - \frac{\sqrt{17}}{4}$$

```
> a:=sol[2];b:=sol[1];
```

$$a := -\frac{1}{4} - \frac{\sqrt{17}}{4}$$

$$b := \frac{1}{4} + \frac{\sqrt{17}}{4}$$

```
> delta:=3+y;
```

$$\delta := 3 + y$$

```
> M:=Muint(delta, y=f..g, x=a..b); M:=value(%); evalf(%);
```

$$M := \int_{-\frac{1}{4} - \frac{\sqrt{17}}{4}}^{\frac{1}{4} + \frac{\sqrt{17}}{4}} \int_{2x^2 - 2}^{|x|} 3 + y \, dy \, dx$$

$$M := \frac{33}{10} + \frac{17\sqrt{17}}{10}$$

10.30927956

```
> ymom:=Muint(y*delta, y=f..g, x=a..b); ymom:=value(%); evalf(%);
```

$$ymom := \int_{-1/4 - \frac{\sqrt{17}}{4}}^{1/4 + \frac{\sqrt{17}}{4}} \int_{2x^2 - 2}^{|x|} y(3+y) dy dx$$

$$ymom := \frac{4999}{6720} - \frac{4369\sqrt{17}}{6720}$$

-1.936733404

```
> ybar:=ymom/M; evalf(%);
```

$$ybar := \frac{\frac{4999}{6720} - \frac{4369\sqrt{17}}{6720}}{\frac{33}{10} + \frac{17\sqrt{17}}{10}}$$

-0.1878631181