1-12	/60
13	/10
14	/10
15	/10
16	/10
Total	/100

Name________________________ ID____________________

MATH 221/253H Exam 1 Fall 2012

Sections 502,201,202 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the area of the triangle whose vertices are
$P = (2,4,-3)$, $Q = (3,4,-2)$ and $R = (0,6,-3)$.

a. 12

b. 6

c. $2\sqrt{3}$

d. $\sqrt{3}$

e. 1

2. Which of the following is the hyperplane in $\mathbb{R}^4$ which contains the point $P = (x,y,z,w) = (8,4,2,1)$ and is tangent to the vectors $\vec{p} = \langle 2,1,0,0\rangle$ $\vec{q} = \langle 0,2,1,0\rangle$ and $\vec{r} = \langle 0,0,2,1\rangle$?
HINT: What is the vector perpendicular to the hyperplane?

a. $8x + 4y + 2z + w = 85$

b. $8x - 4y + 2z - w = 51$

c. $x + 2y + 4z + 8w = 32$

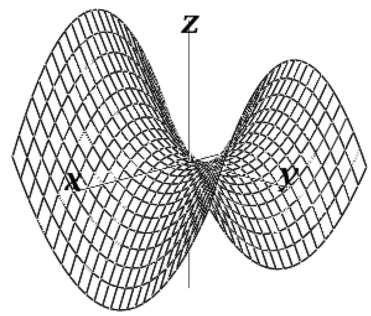
d. $x - 2y + 4z - 8w = 0$

e. $x - y + z - w = 5$

3. The quadratic surface $x^2 + y^2 - z^2 + 4x + 4y - 6z = 0$ is

a. a cone.
b. an elliptic hyperboloid
c. a hyperbolic paraboloid
d. a hyperboloid of 1 sheet
e. a hyperboloid of 2 sheets

4. The plot at the right is the graph of which equation?



a. $z = x^2 - y^2$
b. $z = -x^2 + y^2$
c. $z^2 = x^2 - y^2$
d. $z^2 = -x^2 + y^2$
e. $z^2 - x^2 - y^2 = 1$

5. An airplane is travelling due South with constant speed and constant altitude as it flies over College Station. Since its path is part of a circle around the earth, its acceleration points directly toward the center of the earth. In which direction does it binormal $\hat{B}$ point?

a. Up

b. North

c. South

d. East

e. West

6. For the curve $\vec{r}(t) = (4\cos t, \quad 3t, \quad 4\sin t)$ which of the following is FALSE?

a. $\vec{v} = \langle -4\sin t, \quad 3, \quad 4\cos t\rangle$

b. $\vec{a} = \langle -4\cos t, \quad 0, \quad -4\sin t\rangle$

c. $|\vec{v}| = 25$

d. Arc length between $t = 0$ and $t = 2\pi$ is 10π

e. $a_T = 0$

7. A wire in the shape of the curve $\vec{r}(t) = (4\cos t, \quad 3t, \quad 4\sin t)$ has linear mass density $\rho = y + z$. Find its total mass between $t = 0$ and $t = 2\pi$.

a. $30\pi^2$

b. $6\pi^2$

c. 30π

d. 12π

e. 6π

8. Find the work done to move an object along the curve $\vec{r}(t) = (4\cos t, \quad 3t, \quad 4\sin t)$ between $t = 0$ and $t = 2\pi$ by the force $\vec{F} = \langle z, 0, -x\rangle$?

a. 32π

b. 25π

c. $-25\pi^2$

d. -25π

e. -32π

9. Find the plane tangent to the graph of $z = xe^y$ at the point $(2,0)$. Its z-intercept is

a. $-e$

b. -2

c. 0

d. 2

e. e

10. Find the plane tangent to the graph of $xz^3 + zy^2 + yx^4 = 42$ at the point $(1,2,0)$. Its z-intercept is

a. $\frac{2}{5}$

b. $\frac{4}{5}$

c. $\frac{5}{4}$

d. $\frac{5}{2}$

e. 10

11. Hans Duo is currently at $(x, y, z) = (3, 2, 1)$ and flying the Milenium Eagle through a deadly polaron field whose density is $\rho = x^2 z + yz^2$. In what unit vector direction should he travel to **reduce** the density as fast as possible?

a. $\langle -6, -1, -13 \rangle$

b. $\frac{1}{\sqrt{206}} \langle -6, -1, -13 \rangle$

c. $\langle 6, 1, 13 \rangle$

d. $\frac{1}{\sqrt{206}} \langle -6, 1, -13 \rangle$

e. $\frac{1}{\sqrt{206}} \langle 6, -1, 13 \rangle$

12. The point $(x, y) = (9, 3)$ is a critical point of the function $f(x, y) = x^2 - 2xy^2 + 4y^3$. Use the Second Derivative Test to classify this critical point.

a. local minimum

b. local maximum

c. saddle point

d. TEST FAILS

Work Out: (10 points each. Part credit possible. Show all work.)

13. Find the scalar and vector projections of the vector $\vec{a} = \langle 2, -1, 2 \rangle$ along the vector $\vec{b} = \langle 1, 2, -2 \rangle$.

14. The pressure, P, volume, V, and temperature, T, of an ideal gas are related by

$P = \frac{kT}{V}$ for some constant k. For a certain sample $k = 10 \frac{\text{cm}^3\text{-atm}}{°\text{K}}$.

At a certain instant, the volume and temperature are $V = 2000\,\text{cm}^3$, and $T = 300\,°\text{K}$,

and are increasing at $\frac{dV}{dt} = 40 \frac{\text{cm}^3}{\text{sec}}$, and $\frac{dT}{dt} = 5 \frac{°\text{K}}{\text{sec}}$.

At that instant, what is the pressure, is it increasing or decreasing and at what rate?

15. If two resistors, with resistances R_1 and R_2, are arranged in parallel, the total resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{or} \qquad R = \frac{R_1 R_2}{R_1 + R_2}$$

If $R_1 = 4\Omega$ and $R_2 = 6\Omega$ and the uncertainty in the measurement of R_1 is $\Delta R_1 = 0.03\Omega$ and for R_2 is $\Delta R_2 = 0.02\Omega$. Find R and use differentials to estimate the uncertainty in the measurment of R.

16. Find the point(s) on the surface $z^2 = 46 - 4x - 2y$ which are closest to the origin.
HINT: Explain why you can minimize the square of the distance instead of the distance.
Use the Second Derivative Test to check it is a local minimum.