Name______________________________ ID______________________

| | |
|---|---|
| 1-9 | /45 |
| 10 | /15 |
| 11 | /25 |
| 12 | /15 |
| Total | /100 |

MATH 253H Final Exam Fall 2012

Sections 201-202 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

**1**. Points $A$, $B$, $C$ and $D$ are the vertices of a parallelogram traversed in order. If $A = (2,4,1)$, $B = (3,-2,0)$ and $D = (1,3,-2)$, then $C =$

**a**. $(2,-3,-3)$

**b**. $(4,-1,3)$

**c**. $(0,9,-1)$

**d**. $(6,5,-1)$

**e**. $\left(4,\frac{9}{2},0\right)$

**2**. Which vector is perpendicular to the surface $x^2z^3 + y^3z^2 = 1$ at the point $(3,-2,1)$?

**a**. $(12,-24,43)$

**b**. $(6,-12,43)$

**c**. $(6,12,43)$

**d**. $(6,-12,11)$

**e**. $(-12,-24,-22)$

**3**. Find the point on the elliptic paraboloid $\vec{R}(t,\theta) = (3t\cos\theta, 2t\sin\theta, 1+t^2)$ where a unit normal is $\hat{N} = \left(\frac{-2\sqrt{3}}{5}, \frac{-2}{5}, \frac{3}{5}\right)$.

**a.** $\left(\frac{3}{2}, \sqrt{3}, 2\right)$
**b.** $\left(-\frac{3}{2}\sqrt{3}, -1, 2\right)$
**c.** $\left(3, 2\sqrt{3}, 5\right)$
**d.** $\left(3\sqrt{3}, 2, 5\right)$
**e.** $\left(-3, -2\sqrt{3}, 5\right)$

**4**. Compute $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$

HINT: Plot the region of integration and convert to polar coordinates.

**a.** $\frac{\pi}{2}\ln 10$
**b.** $\pi \ln 10$
**c.** $\frac{\pi}{2}\arctan 3$
**d.** $\pi \arctan 3$
**e.** $\pi \arctan 10$

**5.** Find the mass of a wire in the shape of the curve $\vec{r}(t) = \left(e^t, \sqrt{2}\,t, e^{-t}\right)$ for $-1 \le t \le 1$ if the density is $\rho = x$.

**a.** $\dfrac{e^2}{2} + \dfrac{e^{-2}}{2}$

**b.** $\dfrac{e^2}{2} - \dfrac{e^{-2}}{2}$

**c.** $\dfrac{e^2}{2} - \dfrac{e^{-2}}{2} + 2$

**d.** $e^2 - e^{-2}$

**e.** $e^2 - e^{-2} + 2$

**6.** Find the plane tangent to graph of $z = x\cos y + \sin y$ at $(2, \pi)$.
What is the $z$-intercept?

**a.** $-4 + \pi$

**b.** $4 + \pi$

**c.** $-4 - \pi$

**d.** $4 - \pi$

**e.** $\pi$

**7**. Let $L = \lim_{(x,y)\to(0,0)} \frac{e^{(x^2+y^2)} - 1}{x^2 + y^2}$

**a**. $L$ does not exist by looking at the paths $y = x$ and $y = -x$.
**b**. $L$ exists and $L = 1$ by looking at the paths $y = mx$.
**c**. $L$ does not exist by looking at polar coordinates.
**d**. $L$ exists and $L = 1$ by looking at polar coordinates.
**e**. $L$ exists and $L = 0$ by looking at polar coordinates.

**8**. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (y^2, 4xy)$ along the piece of the parabola $y = x^2$ from $(-2,4)$ to $(2,4)$ followed by the line segment from $(2,4)$ back to $(-2,4)$.
HINT: Use Green's Theorem.

**a**. $\frac{256}{5}$
**b**. $\frac{768}{5}$
**c**. $\frac{64}{3}$
**d**. $\frac{128}{3}$
**e**. $0$

**9**. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (4xy^2, 4x^2y)$ along the line segment from $(1,2)$ to $(3,1)$.
HINT: Find a scalar potential.

**a**. $4$
**b**. $10$
**c**. $20$
**d**. $24$
**e**. $26$

Work Out: (Points indicated. Part credit possible. Show all work.)

**10**. (15 points) A $166$ cm piece of wire is cut into 3 pieces of lengths $a$, $b$ and $c$.

The piece of length $a$ is folded into a square of side $s = \frac{a}{4}$.

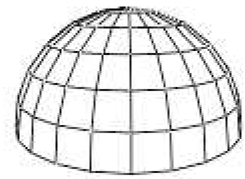
The piece of length $b$ is folded into a rectangle of length $L_1 = \frac{b}{3}$ and width $W_1 = \frac{b}{6}$.

The piece of length $c$ is folded into a rectangle of length $L_2 = \frac{3c}{8}$ and width $W_2 = \frac{c}{8}$.

Find $a$, $b$ and $c$ so that the total area is a minimum.

What is the total area?

**11.** (25 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (xz, yz, x^2 + y^2)$ and the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$.

Be careful with orientations. Use the following steps:

**First the Left Hand Side**:

**a.** Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

**b.** Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $dV =$

**c.** Compute the left hand side:

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

**Second the Right Hand Side**:

The boundary surface consists of a hemisphere $H$ and a disk $D$ with appropriate orientations.

**d.** Parametrize the disk $D$:

$\vec{R}(r,\theta) = \big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\big)$

**e.** Compute the tangent vectors:

$\vec{e}_r = \big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\big)$

$\vec{e}_\theta = \big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\big)$

**f.** Compute the normal vector:

$\vec{N} =$

**g.** Evaluate $\vec{F} = (xz, yz, x^2 + y^2)$ on the disk:

$\vec{F}\big|_{\vec{R}(r,\theta)} =$

**h.** Compute the dot product:

$\vec{F} \cdot \vec{N} =$

**i.** Compute the flux through $D$:

$\iint\limits_D \vec{F} \cdot d\vec{S} =$

**j**. Parametrize the hemisphere $H$:

$\vec{R}(\varphi,\theta) = \Big(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\Big)$

**k**. Compute the tangent vectors:

$\vec{e}_\varphi = \Big(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\Big)$

$\vec{e}_\theta = \Big(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\Big)$

**l**. Compute the normal vector:

$\vec{N} =$

**m**. Evaluate $\vec{F} = (xz, yz, x^2 + y^2)$ on the hemisphere:

$\vec{F}\big|_{\vec{R}(\theta,\varphi)} =$

**n**. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

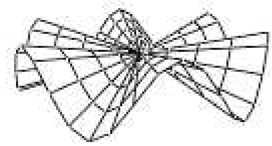
**o**. Compute the flux through $H$:

$\iint\limits_C \vec{F} \cdot d\vec{S} =$

**p**. Compute the **TOTAL** right hand side:

$\iint\limits_{\partial V} \vec{F} \cdot d\vec{S} =$

**12**. (15 points) Compute $\int\int \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-y, x, z)$



over the "clam shell" surface parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r\sin(6\theta))$$

for $r \le 2$ oriented upward.

HINTS: Use Stokes Theorem.

What is the value of $r$ on the boundary?