1-12	/60
13	/20
14	/20
Total	/100

Name________________________

MATH 253 Exam 1 Fall 2016

Sections 201/202 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the distance from the point $\langle 3,4,12\rangle$ to the sphere $x^2+y^2+z^2=64$.

a. 1

b. 5

c. 8

d. 13

e. 105

2. Find a and b so that $a(1,2)+b(2,1)=(0,3)$. What is $a+b$?

a. 1

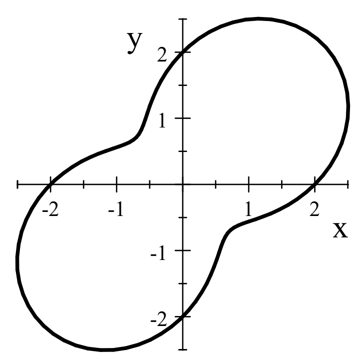
b. 2

c. 3

d. 4

e. 5

3. The plot at the right is which polar curve?



a. $r=2-\cos(2\theta)$

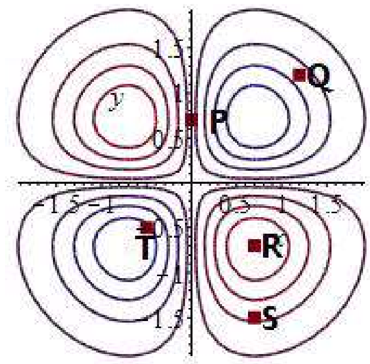
b. $r=2+\cos(2\theta)$

c. $r=2-\sin(2\theta)$

d. $r=2+\sin(2\theta)$

e. $r=\theta$

4. In the plot at the right, which point could be a local maximum?



a. $P = \left(0, \frac{1}{\sqrt{2}}\right)$

b. $Q = \left(\sqrt{2}, \sqrt{2}\right)$

c. $R = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

d. $S = \left(\frac{1}{\sqrt{2}}, -\sqrt{2}\right)$

e. $T = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

5. Find a vector perpendicular to the plane thru the points $P = (2,3,0)$, $Q = (4,-1,-1)$ and $R = (2,0,2)$.

a. $\langle 11,-4,-6 \rangle$

b. $\langle -11,3,-2 \rangle$

c. $\langle -11,-4,-6 \rangle$

d. $\langle -11,-3,-2 \rangle$

e. $\langle -11,4,-6 \rangle$

6. A triangle has vertices at $P = (1,0,4)$, $Q = (1,0,2)$ and $R = \left(2, \sqrt{3}, 0\right)$. Find the angle at Q.

a. $30°$

b. $45°$

c. $60°$

d. $120°$

e. $135°$

7. Find the intersection of the line $\frac{x-2}{-2} = \frac{y-1}{3} = \frac{z+2}{1}$ and the plane $2x + y - z = 3$. At this point $x + y + z =$

- **a**. 1
- **b**. 3
- **c**. 5
- **d**. 7
- **e**. 9

8. Find the plane tangent to the graph of the function $z = f(x,y) = x^2\sin(y) + x\cos(y)$ at the point $(x,y) = (2,\pi)$. Its z-intercept is

- **a**. 4π
- **b**. 2π
- **c**. 2
- **d**. -4π
- **e**. -2π

9. A plane is flying from WEST to EAST, directly over the equator at a constant altitude of 100 kilometers above sea level. (Since the Earth is a sphere, the path of the plane is part of a great circle.) In what direction do $\hat{N}$ and $\hat{B}$ point?

- **a**. $\hat{N}$ points SOUTH and $\hat{B}$ points DOWN
- **b**. $\hat{N}$ points SOUTH and $\hat{B}$ points UP
- **c**. $\hat{N}$ points DOWN and $\hat{B}$ points NORTH
- **d**. $\hat{N}$ points DOWN and $\hat{B}$ points SOUTH
- **e**. $\hat{N}$ points UP and $\hat{B}$ points NORTH

10. Find the mass of a wire in the shape of the semi-circle $\vec{r}(\theta) = (3\cos\theta, 3\sin\theta)$ for $0 \le \theta \le \pi$ if the linear density is given by $\delta = y$.

a. π

b. 3π

c. 6

d. 12

e. 18

11. A bead is pushed along a wire in the shape of the twisted cubic $\vec{r}(t) = (t^2, t^3, t)$ by the force $\vec{F} = \langle x, z, -y \rangle$ from $(1,1,1)$ to $(4,8,2)$. Find the work done.

a. 15

b. 16

c. $\frac{45}{2}$

d. 45

e. 48

12. Compute $\lim\limits_{h \to 0} \dfrac{\sin^3(2x + 2h + 3y) - \sin^3(2x + 3y)}{h}$

a. $6\sin^2(2x + 3y)\cos(2x + 3y)$

b. $6\cos^2(2x + 3y)$

c. $9\sin^2(2x + 3y)\cos(2x + 3y)$

d. $9\cos^2(2x + 3y)$

e. $6\sin^2(2x + 3y)$

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (20 points) As Duke Skywater flies the Century Eagle through the galaxy he wants to maximize the Power of the Force which is given by $F = \frac{3}{D}$ where D is the dark matter density given by $D = x^2 + y^2 + z^2$. His current position is $\vec{r} = (1,2,2)$.

a. If his current velocity is $\vec{v} = (0.3, 0.5, 0.7)$, what is the current rate of change of the Power of the Force, $\frac{dF}{dt}$? (Plug in numbers but you don't need to simplify.)

b. If he wants to change his velocity to increase the Power of the Force as fast as possible, in what **unit** vector direction should he travel?

14. (20 points) For each limit, prove it exists or does not exist. If it exists, find the limit.

a. $$\lim_{(x,y)\to(0,0)} \frac{xy^3}{(x+y^3)^2}$$

b. $$\lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{x^2+y^2}$$