1-8	/40
9	/24
10	/10⁺
11	/13
12	/13

Name____________________ ID____________ Section______

MATH 253 Honors EXAM 1 Spring 1999

Sections 201-203 P. Yasskin

Multiple Choice: (5 points each)

Problems 1 – 4: Consider the vectors: $\vec{a} = (2,0,-2)$, $\vec{b} = (0,-3,3)$ and $\vec{c} = (1,1,1)$.

1. The angle between $\vec{a}$ and $\vec{b}$ is

a. $30°$
b. $45°$
c. $90°$
d. $120°$
e. $150°$

2. The vector projection of $\vec{a}$ along $\vec{b}$ is

a. $(0,1,-1)$
b. $(0,-1,1)$
c. $(-2,-3,5)$
d. $\left(0,3\sqrt{2},-3\sqrt{2}\right)$
e. $\left(0,-3\sqrt{2},3\sqrt{2}\right)$

3. The area of a triangle with $\vec{a}$ and $\vec{b}$ as two sides is

a. 3
b. 6
c. $3\sqrt{3}$
d. $6\sqrt{3}$
e. 54

4. The volume of the parallelepiped with edges $\vec{a}$, $\vec{b}$ and $\vec{c}$ is

a. -18
b. -6
c. 3
d. 6
e. 18

Problems 5 – 7: The pressure in an ideal gas is given by $P = k\rho T$ where k is a constant, ρ is the density and T is the temperature. The pressure, density and temperature are all functions of position. At the point $Q = (1,2,3)$, the density is $\rho(Q) = 1.5$ and its gradient is $\vec{\nabla}\rho(Q) = (.2,.3,-.1)$. Also at that point, the temperature is $T(Q) = 24$ and its gradient is $\vec{\nabla}T(Q) = (-3,1,2)$.

5. At the point Q, the pressure is $P(Q) = 36k$. What is the gradient of the pressure?

a. $\vec{\nabla}P(Q) = k(.7,-.1,1.1)$
b. $\vec{\nabla}P(Q) = k(.3,8.7,.6)$
c. $\vec{\nabla}P(Q) = k(.3,-8.7,.6)$
d. $\vec{\nabla}P(Q) = k(-2.8,1.3,1.9)$
e. $\vec{\nabla}P(Q) = k(-2.8,-1.3,1.9)$

6. If a fly is located at the point Q, in what direction should the fly travel to **cool off** as soon as possible?

a. $(-.2,-.3,.1)$
b. $(3,-1,-2)$
c. $(-3,1,2)$
d. $(2,-1,3)$
e. $(2,1,3)$

7. If a fly is located at the point Q and travelling with velocity $\vec{v} = (3,4,12)$, how fast is the density changing at the location of the fly?

a. $\frac{d\rho}{dt}(Q) = -7.8$
b. $\frac{d\rho}{dt}(Q) = -.6$
c. $\frac{d\rho}{dt}(Q) = \frac{.6}{13}$
d. $\frac{d\rho}{dt}(Q) = .6$
e. $\frac{d\rho}{dt}(Q) = 7.8$

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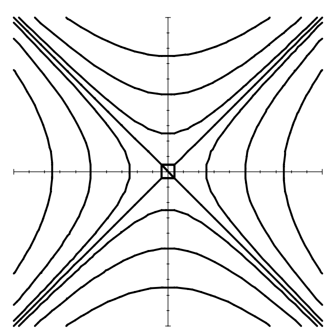
8. The graph at the right is the contour plot of which function?



a. $y^2 - x^2$
b. xy
c. $x^2 + y^2$
d. $y - x^2$
e. $x - y^2$

9. (24 points) Consider the parametric curve $\vec{r}(t) = (t^3, 3t^2, 6t)$.

a. Compute the velocity and acceleration:

$\vec{v} =$

$\vec{a} =$

b. Find a parametric equation for the line tangent to the curve at $t = 1$.

c. Find a non-parametric (symmetric) equation for the line tangent to the curve at $t = 1$.

d. Find a parametric equation for the plane instantaneously containing the curve at $t = 1$.

e. Find a non-parametric equation for the plane instantaneously containing the curve at $t = 1$.

f. Find the arclength of the curve between $t = 0$ and $t = 2$.

10. (10 points) Does each limit exist? Why or why not? Find the value of the one that exists. (***Up to* 4 *points extra credit for a good explanation***.)

a. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2 + 2y^2}$

b. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{\sqrt{x^2 + 2y^2}}$

11. (13 points) Find the equation of the plane tangent to the graph of the function $f(x,y) = 3x\sin y - 2y\cos x$ at the point $(x,y) = \left(0, \frac{\pi}{2}\right)$.

12. (13 points) Find the equation of the plane tangent to the surface $F(x,y,z) = x^2y + y^3z + z^4x = 29$ at the point $P = (x,y,z) = (3,2,1)$.