Name________________________ ID____________ Section______

1-6	/30
7	/20
8	/30
9	/20

MATH 253 Honors EXAM 3 Spring 1999

Sections 201-203 P. Yasskin

Multiple Choice: (5 points each)

1. If $F = (xz, yx, zy)$ then $\vec{\nabla} \bullet \vec{F} =$

- **a**. $z - x + y$
- **b**. $(z, -x, y)$
- **c**. $x + y + z$
- **d**. (z, x, y)
- **e**. $-x + y - z$

2. If $F = (xz, yx, zy)$ then $\vec{\nabla} \times \vec{F} =$

- **a**. $z - x + y$
- **b**. $(z, -x, y)$
- **c**. $x + y + z$
- **d**. (z, x, y)
- **e**. $-x + y - z$

3. If $F = (\frac{x \sin z}{x^2 + y^2}, \frac{y \cos x}{x^2 + y^2}, \tan(zy))$ then $\vec{\nabla} \bullet \vec{\nabla} \times \vec{F} =$

- **a**. $\frac{\sin z - \cos x}{(x^2 + y^2)^2} + 2\sec(zy)\tan(zy))$
- **b**. $\frac{\sin z - \cos x}{(x^2 + y^2)^2} + 2\sec(zy)\tan(zy))$
- **c**. $\frac{\sin z - \cos x}{(x^2 + y^2)^3} + 2y\sec(zy)\tan(zy)) + \sec^2(zy)$
- **d**. $\frac{\cos z - \sin x}{(x^2 + y^2)^3} + 2y\sec(zy)\tan(zy)) + \sec^2(zy)$
- **e**. None of These

4. Compute the line integral $\int y\,dx - x\,dy$ counterclockwise around the semicircle $x^2 + y^2 = 9$ from $(3,0)$ to $(-3,0)$.   (HINT: Parametrize the curve.)

a. -9π
b. -2π
c. π
d. 2π
e. 9π

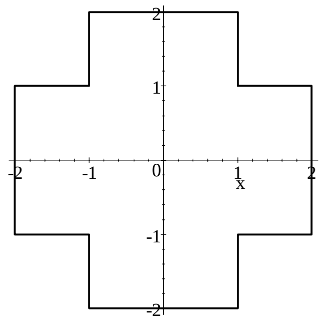
5. Compute the line integral $\int \vec{F} \bullet d\vec{s}$ for the vector field $\vec{F} = \left(\frac{1}{x}, \frac{1}{y}\right)$ along the curve $\vec{r}(t) = \left(e^{\cos(t^2)}, e^{\sin(t^2)}\right)$ for $0 \le t \le \sqrt{\pi}$.   (HINT: Find a potential.)

a. -2
b. 0
c. $\frac{2}{e}$
d. 1
e. π

6. Compute $\iint_{\partial C} \vec{F} \bullet d\vec{S}$ for the vector field $\vec{F} = \left(zx^3, zy^3, z^2(x^2 + y^2)\right)$ over the complete surface of the solid cylinder $C = \{(x,y,z) \mid x^2 + y^2 \le 4,\ 0 \le z \le 3\}$ with normal pointing outward.

a. 360π
b. 180π
c. $90\pi 1$
d. 60π
e. 30π

7. (20 points) Compute the line integral $\oint \frac{y}{x^2+y^2}\,dx - \frac{x}{x^2+y^2}\,dy$ counterclockwise around the boundary of the plus sign shown below.
Be sure to justify any theorem you use. (Hint: The answer is not zero.)



8. (30 points) Stokes' Theorem states that if S is a surface in 3-space and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S then

$$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint_{\partial S} \vec{F} \bullet d\vec{s}$$

Verify Stokes' Theorem if $F = (yx^2, -xy^2, x^2z + y^2z)$ and S is the hemisphere $z = \sqrt{4 - x^2 - y^2}$ with **normal pointing up and out**.

8a. (10 points) Compute $\oint_{\partial S} \vec{F} \bullet d\vec{s}$ using the following steps: (Remember to check the orientation of the curve.)

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}\left(\vec{r}(\theta)\right) =$

$\oint_{\partial S} \vec{F} \bullet d\vec{s} =$

8b. (5 points) Compute $\vec{\nabla} \times \vec{F}$. (HINT: Use rectangular coordinates.)

$\vec{\nabla} \times \vec{F} =$

8c. (15 points) Compute $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$ using the following steps:

Recall $F = (yx^2, -xy^2, x^2z + y^2z)$ and S is the hemisphere $z = \sqrt{4 - x^2 - y^2}$ with **normal pointing up and out**.

$\vec{R}(\theta, \phi) =$

$\vec{R}_\theta =$

$\vec{R}_\phi =$

$\vec{N} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(\theta, \phi)\right) =$

$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} =$

9. (20 points)

The spider web at the right is the graph of the hyperbolic paraboloid $z = xy$.

It may be parametrized as

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2\sin\theta\cos\theta)$.

Find the area of the web for $r \leq \sqrt{8}$.