1-10	/50
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Name______________________ ID____________ Section______

MATH 253 Honors FINAL EXAM Spring 1999

Sections 201-203 P. Yasskin

Multiple Choice: (5 points each)

1. Find the volume of the parallelepiped with edges $(3,2,0)$, $(-1,1,2)$ and $(0,4,1)$.

a. -23
b. -19
c. 19
d. 21
e. 23

2. Find the unit tangent vector $\hat{T}$ to the curve $\vec{r}(t) = (3t, 2t^2, 4t^3)$ at the point $\vec{r}(1) = (3,2,4)$.

a. $\left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$
b. $\left(\frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{12}{\sqrt{29}}\right)$
c. $\left(\frac{3}{169}, \frac{4}{169}, \frac{12}{169}\right)$
d. $\left(\frac{3}{29}, \frac{4}{29}, \frac{12}{29}\right)$
e. $\left(\frac{3}{169}, \frac{-4}{169}, \frac{12}{169}\right)$

3. If a jet flies around the world from East to West, directly above the equator, in what direction does the unit binormal $\hat{B}$ point?

- **a.** North
- **b.** South
- **c.** East
- **d.** West
- **e.** Down (toward the center of the earth)

4. At the point (x,y,z) where the line $\vec{r}(t) = (2+t, 3-t, t)$ intersects the plane $2x - y + z = 5,$ we have $x + y + z =$

- **a.** 2
- **b.** 3
- **c.** 4
- **d.** 5
- **e.** 6

5. The temperature in an ideal gas is given by $T = \kappa\frac{P}{\rho}$ where κ is a constant, P is the pressure and ρ is the density. At a certain point $Q = (1,2,3),$ we have

$$P(Q) = 4 \qquad \vec{\nabla}P(Q) = (-3,2,1)$$

$$\rho(Q) = 2 \qquad \vec{\nabla}\rho(Q) = (3,-1,2)$$

So at the point Q, the temperature is $T(Q) = 2\kappa$ and its gradient is $\vec{\nabla}T(Q) =$

- **a.** $\kappa(-4.5, 0, 2.5)$
- **b.** $\kappa(1.5, 0, 2.5)$
- **c.** $\kappa(1.5, 2, -4.5)$
- **d.** $\kappa(-4.5, 2, -1.5)$
- **e.** $\kappa(-1.5, 2, 2.5)$

6. The saddle surface $z = xy$ may be parametrized as $\vec{R}(u,v) = (u,v,uv)$. Find the plane tangent to the surface at the point $(1,2,2)$.

- **a.** $3x + y - z = 3$
- **b.** $2x + y - z = 2$
- **c.** $3x + 2y - z = 5$
- **d.** $2x - y + z = 2$
- **e.** $3x - y + z = 3$

7. Find the minimum value of the function $f = x^2 + y^2 + z^2$ on the plane $x + 2y + 3z = 14$.

- **a.** 0
- **b.** $\frac{7}{4}$
- **c.** $\frac{7}{2}$
- **d.** 14
- **e.** 28

8. Compute $\int_0^3 \int_{y^2}^9 y\cos(x^2)\ dx\ dy$

a. $\frac{1}{4}\sin 81$
b. $\frac{1}{2}\cos 9 - \frac{1}{2}$
c. $\frac{9}{2}\sin 81 + \cos 9 - 1$
d. $-\frac{9}{2}\sin 81 + \frac{9}{2}\sin y^4$
e. $\frac{9}{2}\sin 81 - \cos 9 + 1$

9. Compute $\iiint z^2\ dV$ over the solid sphere $x^2 + y^2 + z^2 \le 4$.

a. $\frac{64\pi}{5}$
b. $\frac{256\pi}{3}$
c. $\frac{48\pi}{5}$
d. $\frac{64\pi}{15}$
e. $\frac{128\pi}{15}$

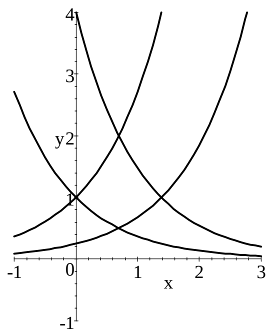
10. Compute $\iint \vec{F} \bullet d\vec{S}$ for $\vec{F} = (x, y^3, z)$ over the surface of the cube $0 \le x \le 1,\quad 0 \le y \le 1,\quad 0 \le z \le 1$ with outward normal.

a. 1
b. 2
c. 3
d. 4
e. 6

11. (15 points) Find the area of the diamond shaped region between the curves

$$y = e^x, \quad y = \frac{1}{4}e^x, \quad y = e^{-x} \quad \text{and} \quad y = 4e^{-x}.$$

You **must** use the curvilinear coordinates $u = ye^{-x}$ and $v = ye^x$.



12. (10 points) Find the mass of a wire in the shape of the curve $y = \ln(\cos x)$ for $0 \le x \le \frac{\pi}{4}$ if the density is $\rho = \frac{\sin x}{e^y}$.

Note: The wire may be parametrized as $\vec{r}(t) = (t, \ln(\cos t))$.

13. (10 points) Compute $\oint x\ dx + z\ dy - y\ dz$ around the boundary of the triangle with vertices $(0,0,0)$, $(0,1,0)$ and $(0,0,1)$, traversed in this order of the vertices. Hint: The yz-plane may be parametrized as $\vec{R}(u,v) = (0,u,v)$.

14. (15 points) Compute $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$

for $\vec{F} = (x^2 y, y, z^2)$ over the piece of the sphere $x^2 + y^2 + z^2 = 25$ for $0 \le z \le 4$ with normal pointing away from the z-axis.

Hint: Parametrize the upper and lower edges.