Name____________________ ID____________ Section______

1-7	/56
8	/12$^+$
9	/16
10	/16

MATH 253 Honors EXAM 1 Fall 1999

Sections 201-202 P. Yasskin

Multiple Choice: (8 points each)

1. Find the volume of the parallelepiped with edges $\vec{a} = (2,4,0)$, $\vec{b} = (-1,0,3)$ and $\vec{c} = (0,1,-2)$.

a. -14
b. -12
c. 2
d. 12
e. 14

2. At the point where the line $\frac{x+4}{3} = \frac{y-1}{2} = \frac{z+2}{4}$ intersects the plane $(x,y,z) = (4-2s+t, 3+2s, 6-t)$ we have $2x+y+z =$

a. 15
b. 13
c. 11
d. 9
e. 7

3. Find the equation of the plane which is perpendicular to the line through $P = (1,3,5)$ and $Q = (3,7,1)$ and is equidistant from P and Q.

a. $x+2y-2z = 12$
b. $x+2y-2z = 6$
c. $x+2y-2z = -6$
d. $x+2y-2z = -3$
e. $2x+4y-4z = 24$

Problems 4 – 6: The temperature in an ideal gas is given by $T = \kappa \frac{P}{\rho}$ where κ is a constant, P is the pressure and ρ is the density. The pressure, density and temperature are all functions of position. At a certain point $Q = (1,2,3)$, we have

$$P(Q) = 4 \qquad \vec{\nabla}P(Q) = (-3,2,1)$$

$$\rho(Q) = 2 \qquad \vec{\nabla}\rho(Q) = (3,-1,2)$$

4. Use the linear approximation to estimate the pressure at the point $R = (1.2, 1.9, 3.1)$.

a. 2.9
b. 3.1
c. 3.3
d. 4.7
e. 7.3

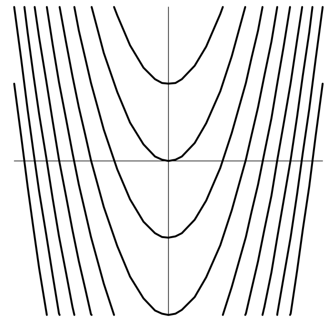
5. At the point Q, the temperature is $T(Q) = 2\kappa$ and its gradient is $\vec{\nabla}T(Q) =$

a. $\kappa(-4.5, 0, 2.5)$
b. $\kappa(1.5, 0, 2.5)$
c. $\kappa(1.5, 2, -4.5)$
d. $\kappa(-4.5, 2, -1.5)$
e. $\kappa(-1.5, 2, 2.5)$

6. If a fly is located at the point Q and travelling with velocity $\vec{v} = (3,4,12)$, how fast is the density changing at the location of the fly?

a. $\frac{d\rho}{dt}(Q) = 37$
b. $\frac{d\rho}{dt}(Q) = 29$
c. $\frac{d\rho}{dt}(Q) = 21$
d. $\frac{d\rho}{dt}(Q) = 2.1$
e. $\frac{d\rho}{dt}(Q) = .21$

7. The graph at the right is the contour plot of which function?



a. $y^2 - x^2$

b. xy

c. $x^2 + y^2$

d. $y - x^2$

e. $x - y^2$

8. (12 points) Does each limit exist? Why or why not? Find the value of the one that exists. (***Up to* 4 *points extra credit for a good explanation***.)

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$

9. (16 points) Consider the parametric curve $\vec{r}(t) = (t, 4e^{t/2}, 2e^{t})$.

a. Compute the velocity and acceleration:

$\vec{v} =$

$\vec{a} =$

b. Find a parametric equation for the line tangent to the curve at $t = 0$.

c. Find a parametric equation for the plane instantaneously containing the curve at $t = 0$. This is the plane containing the velocity and the acceleration.

d. Find a non-parametric equation for the plane instantaneously containing the curve at $t = 0$.

e. Find the arclength of the curve between $t = 0$ and $t = 2$.
HINT: Factor inside the square root.

10. (16 points) Find the equation of the plane tangent to the graph of the function $f(x,y) = 3xe^y - 2x^2e^{2y}$ at the point $(x,y) = (1, \ln 2)$.