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MATH 253 Honors EXAM 3 Fall 1999

Sections 201-202 P. Yasskin

Multiple Choice: (8 points each)

1. Compute the line integral $\int_{A\,H}^{B} \vec{F} \bullet d\vec{s}$ of the vector field $\vec{F} = (yz, -xz, z)$ along the helix H parametrized by $\vec{r}(t) = (3\cos t, 3\sin t, 4t)$ between $A = (3,0,0)$ and $B = (-3,0,4\pi)$.

- **a.** 20π
- **b.** 26π
- **c.** $26\pi^2$
- **d.** -20π
- **e.** $-10\pi^2$

2. Find the total mass of the helix H parametrized by $\vec{r}(t) = (3\cos t, 3\sin t, 4t)$ between $A = (3,0,0)$ and $B = (-3,0,4\pi)$ if the linear mass density is $\rho = z^2$.

- **a.** $\dfrac{16\pi^3}{3}$
- **b.** $\dfrac{40\pi^3}{3}$
- **c.** $\dfrac{80\pi^3}{3}$
- **d.** $16\pi^3$
- **e.** $80\pi^3$

3. Compute $\oint(-xy^2\,dx + x^2y\,dy)$ counterclockwise around the complete boundary of the triangle whose vertices are $(0,0)$, $(2,4)$ and $(0,4)$. HINT: Use Green's Theorem.

- **a.** 4
- **b.** 8
- **c.** 16
- **d.** 32
- **e.** 64

4. Compute $\int(yz\,dx + xz\,dy + xy\,dz)$ along the curve $\vec{r}(t) = \left(e^{\sin 4t}, \cos 5t, \ln\left(1 + \frac{t}{\pi}\right)\right)$ between $t = 0$ and $t = \pi$. HINT: Find a scalar potential for $\vec{F} = (yz, xz, xy)$.

- **a.** $-\ln 2$
- **b.** $1 - \ln 2$
- **c.** $-1 - \ln 2$
- **d.** $1 + \ln 2$
- **e.** $-1 + \ln 2$

5. (25 points) Stokes' Theorem states that if S is a surface in 3-space and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S then

$$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint_{\partial S} \vec{F} \bullet d\vec{s}$$

Verify Stokes' Theorem if

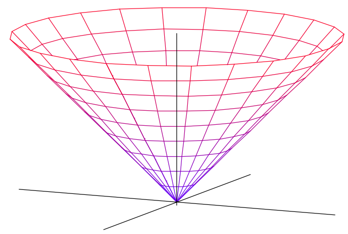
$F = (y, -x, x^2 + y^2)$

and S is the cone $z = \sqrt{x^2 + y^2}$ for $z \le 2$

with **normal pointing up and in**.

The cone may be parametrized by:

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$



5a. (16 points) Compute $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$ using the following steps:

$\vec{\nabla} \times \vec{F} =$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} =$

5b (9 points) Recall $F = (y, -x, x^2 + y^2)$ and S is the cone $z = \sqrt{x^2 + y^2}$ with **normal pointing up and in**. Compute $\oint_{\partial S} \vec{F} \bullet d\vec{s}$ using the following steps:

(Remember to check the orientation of the curve.)

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial S} \vec{F} \bullet d\vec{s} =$

6. (25 points) Gauss' Theorem states that if V is a solid region and ∂V is its boundary surface with **outward normal** then

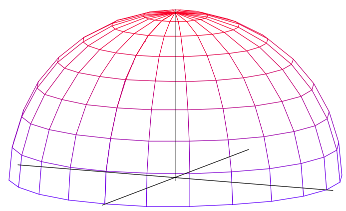
$$\iiint_V \vec{\nabla} \bullet \vec{F}\, dV = \iint_{\partial V} \vec{F} \bullet d\vec{S}$$

Verify Gauss' Theorem if

$F = (xz, yz, x^2 + y^2)$

and V is the solid hemisphere

$0 \le z \le \sqrt{4 - x^2 - y^2}$.



Notice that ∂V consists of two parts:

the hemisphere H: $z = \sqrt{4 - x^2 - y^2}$

and a disk D: $x^2 + y^2 \le 4$ with $z = 0$

6a. (5 pts) Compute $\iiint_V \vec{\nabla} \bullet \vec{F}\, dV$.

HINT: Compute the divergence in rectangular and the integral in spherical.

$\vec{\nabla} \bullet \vec{F} =$

$\iiint_V \vec{\nabla} \bullet \vec{F}\, dV =$

6b. (8 pts) Compute $\iint_D \vec{F} \bullet d\vec{S}$. (HINT: You parametrize the disk.)

$\vec{R}(r,\theta) =$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\vec{F}\left(\vec{R}(r,\theta)\right) =$

$\iint_D \vec{F} \bullet d\vec{S} =$

Recall $F = (xz, yz, x^2 + y^2)$ and V is the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$.

6c. (9 pts) Compute $\iint_H \vec{F} \bullet d\vec{S}$ over the hemisphere parametrized by

$\vec{R}(\varphi, \theta) = (2\sin\varphi\cos\theta, 2\sin\varphi\sin\theta, 2\cos\varphi)$

$\vec{R}_\varphi =$

$\vec{R}_\theta =$

$\vec{N} =$

$\vec{F}\left(\vec{R}(\varphi, \theta)\right) =$

$\vec{F} \bullet \vec{N} =$

$\iint_H \vec{F} \bullet d\vec{S} =$

6d. (3 pts) Combine $\iint_H \vec{F} \bullet d\vec{S}$ and $\iint_D \vec{F} \bullet d\vec{S}$. to obtain $\iint_{\partial V} \vec{F} \bullet d\vec{S}$.

Be sure to discuss the orientations of the surfaces (here or above) and give a formula before you plug in numbers.

$\iint_{\partial V} \vec{F} \bullet d\vec{S} =$

7. (20 points) The paraboloid at the right is the graph of the equation $z = 4x^2 + 4y^2$.

It may be parametrized as

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 4r^2).$$

Find the area of the paraboloid for $z \le 16$.