

Name \_\_\_\_\_ ID \_\_\_\_\_ Section \_\_\_\_\_

MATH 253 Honors  
Sections 201-202

FINAL EXAM

Fall 1999  
P. Yasskin

1-7	/49
8	/15
9	/10
10	/15
11	/15

Multiple Choice: (7 points each)

1. Consider the line through the point  $P = (4, 4, 4)$  which is perpendicular to the plane  $x + 2y + 3z = 7$ . Its tangent vector is
  - a.  $(3, 2, 1)$
  - b.  $(1, 2, 3)$
  - c.  $(7, 6, 5)$
  - d.  $(5, 6, 7)$
  - e.  $(4, 4, 4)$
  
2. Find the plane tangent to the hyperbolic paraboloid  $x - yz = 0$  at the point  $P = (6, 3, 2)$ . Which of the following points does **not** lie on this plane?
  - a.  $(-6, 0, 0)$
  - b.  $(0, 3, 0)$
  - c.  $(0, 0, 2)$
  - d.  $(1, -1, -1)$
  - e.  $(-1, 1, 1)$
  
3. Duke Skywalker is flying the Millennium Eagle through a polaron field. His galactic coordinates are  $(2300, 4200, 1600)$  measured in lightseconds and his velocity is  $\vec{v} = (.2, .3, .4)$  measured in lightseconds per second. He measures the strength of the polaron field is  $p = 274$  milliwookies and its gradient is  $\vec{\nabla}p = (3, 2, 2)$  milliwookies per lightsecond. Assuming a linear approximation for the polaron field and that his velocity is constant, how many seconds will Duke need to wait until the polaron field has grown to 286 milliwookies?
  - a. 2
  - b. 3
  - c. 4
  - d. 6
  - e. 12

4. Consider the surface  $S$  parametrized by  $\vec{R}(u, v) = (u + v, u - v, uv)$  for  $0 \leq u \leq 2$  and  $0 \leq v \leq 4$ . Compute  $\iint_S \vec{F} \cdot d\vec{S}$  where  $\vec{F} = (y, x, y)$ .

- a. -32
- b. -16
- c. 16
- d. 32
- e. 64

5. Consider the surface  $S$  parametrized by  $\vec{R}(u, v) = (u + v, u - v, uv)$ . Find the plane tangent to this surface at the point  $P = \vec{R}(1, 2) = (3, -1, 2)$ . Which of the following points does **not** lie on this plane?

- a. (3, 0, 0)
- b. (0, 4, 0)
- c. (0, 0, -2)
- d. (1, 1, 0)
- e. (0, 6, 1)

6. Compute  $\oint (-x^2y^2 dx + 2xy^3 dy)$  over the complete boundary of the semicircular area  $0 \leq y \leq \sqrt{4 - x^2}$  traversed counterclockwise.

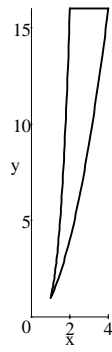
- a. 0
- b. 16
- c.  $\frac{4}{5}$
- d.  $\frac{80}{5}$
- e.  $\frac{128}{5}$

7. Compute  $\iint_S \frac{x^3z^2}{3} dy dz + \frac{y^3z^2}{3} dz dx + \frac{z^5}{5} dx dy$  over the complete surface of the sphere  $x^2 + y^2 + z^2 = 4$  with outward normal.

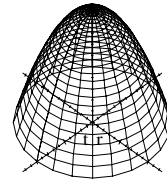
- a.  $\frac{512\pi}{21}$
- b.  $\frac{32\pi^2}{5}$
- c.  $\frac{128\pi}{5}$
- d.  $\frac{16\pi}{3}$
- e.  $\frac{256\pi}{15}$

8. (15 points) Find the point in the first octant on the surface  $z = \frac{32}{x^4 y^2}$  which is closest to the origin.

9. (10 points) Compute  $\iint_R x \, dA$  over the region  $R$  in the first quadrant bounded by the curves  $y = x^2$ ,  $y = x^4$  and  $y = 16$ .



10. (15 points) Find the mass and center of mass of the solid below the paraboloid  $z = 4 - x^2 - y^2$  above the  $xy$ -plane, if the density is  $\delta = x^2 + y^2$ . (11 points for setting up the integrals and the final formula.)



11. (15 points) Find the area and centroid of the **right** leaf of the rose

$$r = 2\cos^2\theta.$$

(12 points for setting up the integrals and the final formula.)

