Name____

Math 304 Exam 1 Version A Spring 2017

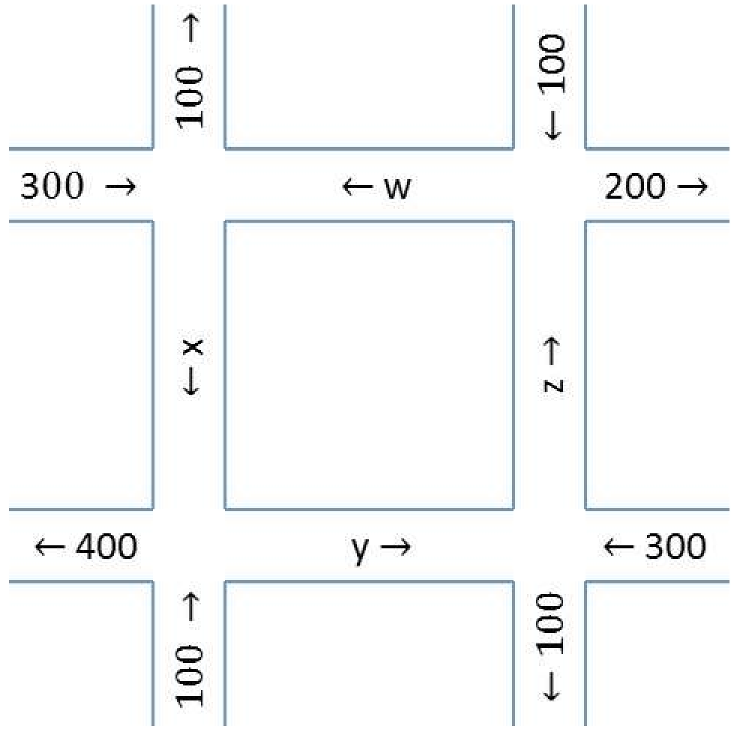
Section 501 P. Yasskin

Points indicated. Show all work.

1	/20	3	/30
2	/45	4	/10
		Total	/105

1. $(20$ points$)$ Consider the traffic flow system shown at the right.

a. Write out the equations for the system.
Write out the augmented matrix.
Keep the variables in the order w, x, y, z.
DO NOT SOLVE THE SYSTEM.



b. Compute the determinant of the matrix of coefficients.

c. One solution is $w = 200$, $x = 400$, $y = 100$, $z = 300$. How many solutions are there?

Circle one:

Exactly 1 solution. Exactly 2 solutions. Exactly 4 solutions. Infinitely many solutions.

2. $\left(45 \text{ points}\right)$ Let $A = \begin{pmatrix} 1 & 2 & 0 & 1 & 0 \\ -1 & -2 & 3 & 8 & 3 \\ 2 & 4 & 0 & 2 & 1 \\ -1 & -2 & 1 & 2 & 1 \end{pmatrix}$.

a. Transform A into reduced row eschelon form. Call the result $rref(A)$. (Be sure to give reasons for each step.)

b. How many leading 1's are there in $rref(A)$? $\#1$'s $=$ ______________

c. What are the dimensions of the **null space**, **column space** and **row space** of A?

$\dim(N(A)) =$ ___________ $\dim(Col(A)) =$ ___________ $\dim(Row(A)) =$ ___________

d. Find a basis for $Col(A)$.

(continued)

e. Find a basis for $Row(A)$.

f. Find a basis for $N(A)$.

3. $\left(30 \text{ points}\right)$ Consider the vector space $P_3 = \left\{\text{polynomials of degree } < 3\right\}$. The standard basis is

$$e_1 = 1 \qquad e_2 = x \qquad e_3 = x^2$$

Let the f basis be

$$f_1 = 1 + x^2 \qquad f_2 = x + x^2 \qquad f_3 = x^2$$

Let the g basis be

$$g_1 = 1 \qquad g_2 = 1 + x \qquad g_3 = 1 + x^2$$

a. Find the change of basis matrix from the f basis to the e basis. Call it $\underset{e \leftarrow f}{C}$.

b. Find the change of basis matrix from the g basis to the e basis. Call it $\underset{e \leftarrow g}{C}$.

c. Find the change of basis matrix from the f basis to the g basis. Call it $\underset{g \leftarrow f}{C}$.

d. Use $\underset{g \leftarrow f}{C}$ to rewrite the polynomial $p = 6f_1 + 3f_2 - 2f_3$ in the g basis, i.e. find a, b, and c so that $p = ag_1 + bg_2 + cg_3$.

4. $(10$ points$)$ By definition, a matrix, A, is idempotent if $A^2 = A$.

a. Show if A is idempotent then $\mathbf{1} - A$ is also idempotent.

b. Show if A is idempotent then $\mathbf{1} + A$ is non-singular and $(\mathbf{1} + A)^{-1} = \mathbf{1} - \frac{1}{2}A$.