

[Math 308 · Maple Quiz 1 · Summer 2003]

[Sec 301,302 · Version A]

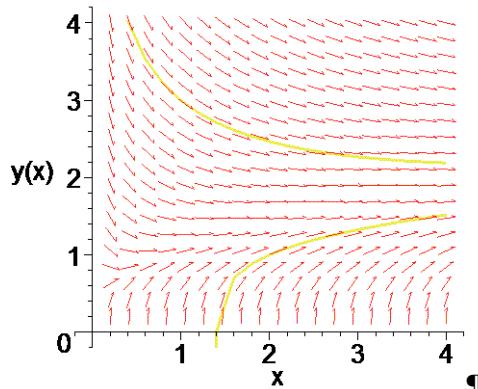
[> restart; with(DEtools):]

[#1]

[> deq:=diff(y(x),x)=(2*x-y(x)^3)/(3*x*y(x)^2);]

$$deq := \frac{d}{dx} y(x) = \frac{1}{3} \frac{2x - y(x)^3}{xy(x)^2}$$

[> DEplot(deq,y(x),x=0..4,y=0..4,[[2,1],[1,3]]);]



[#2]

[> F:=(x,y)->(2*x-y^3)/(3*x*y^2);]

$$F := (x, y) \rightarrow \frac{1}{3} \frac{2x - y^3}{xy^2}$$

[> h:=.2;]

$$h := 0.2$$

[> xs[0]:=2; ys[0]:=1;]

$$xs_0 := 2$$

$$ys_0 := 1$$

[> for i from 1 to 10 do]

[xs[i]:=xs[i-1]+h;]

[ys[i]:=ys[i-1]+F(xs[i-1],ys[i-1])*h;]

[end do;]

$$xs_1 := 2.2$$

$$ys_1 := 1.100000000$$

$$xs_2 := 2.4$$

$$ys_2 := 1.176859504$$

$$xs_3 := 2.6$$

$$ys_3 := 1.240438636$$

$$xs_4 := 2.8$$

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 $ys_4 := 1.295286428$ 
 $xs_5 := 3.0$ 
 $ys_5 := 1.343916987$ 
 $xs_6 := 3.2$ 
 $ys_6 := 1.387875532$ 
 $xs_7 := 3.4$ 
 $ys_7 := 1.428182431$ 
 $xs_8 := 3.6$ 
 $ys_8 := 1.465547782$ 
 $xs_9 := 3.8$ 
 $ys_9 := 1.500486159$ 
 $xs_{10} := 4.0$ 
 $ys_{10} := 1.533382706$ 

#3
> init:=y(2)=1;
 $init := y(2) = 1$ 
> sol:=dsolve({deq, init}, y(x));
 $sol := y(x) = \frac{((x^2 - 2)x^2)^{(1/3)}}{x}$ 
> plot(rhs(sol), x=0..4, y=0..4);

> subs(x=4, rhs(sol));
 $\text{evalf}(\%)$ 
 $\frac{224}{4}^{(1/3)}$ 
 $1.518294486$ 

```

#4

The Euler approximation 1.533 is bigger than the exact value 1.518 because the solution is concave down so the tangent lines are always above the curve.