Name_____ NetID_____

MATH 308   Exam 2   Spring 2009

Section 511   Hand Computations   P. Yasskin

1	/10	4	/30
2	/10	5	/30
3	/10	6	/15
Total			/105

1. (10 points)   Tank X initially contains $250\,\text{L}$ of sugar water with concentration 0.3 gm/L.
Tank Y initially contains $500\,\text{L}$ of sugar water with concentration 0.2 gm/L.
Sugar water with concentration 0.4 gm/L   is flowing into tank X at 10 L/min.
Pure water is flowing into tank Y at 5 L/min.
Sugar water is pumped from tank X to tank Y at 25 L/min.
Sugar water is pumped from tank Y to tank X at 15 L/min.
Finally, sugar water is draining from tank Y at 15 L/min.

Draw a **figure**. Define your **variables**.
Set up the **differential equations** and **initial conditions**.
Do not solve the equations.

2. (10 points)   Consider the mass and spring system shown in the figure.

k1   b1   k2   b2   k3

m1   m2
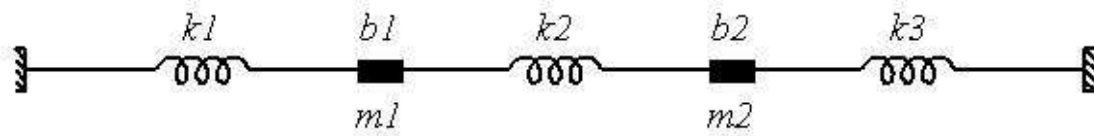
The masses are $m_1 = 2$ kg   and $m_2 = 3$ kg.
The spring constants are $k_1 = 4$ N/m, $k_2 = 5$ N/m   and $k_3 = 6$ N/m.
The drag coefficients are $b_1 = 7$ N·sec/m   and $b_2 = 8$ N·sec/m.
Initially, mass m_1 is moved   2 m   to the left and given a velocity of   4 m/sec   to the right,
while mass m_2 is moved   3 m   to the right and given a velocity of   5 m/sec   to the left.

Define your **variables**. Set up the **differential equations** and **initial conditions**.
Do not solve the equations.

3. (10 points)   Consider the circuit shown.
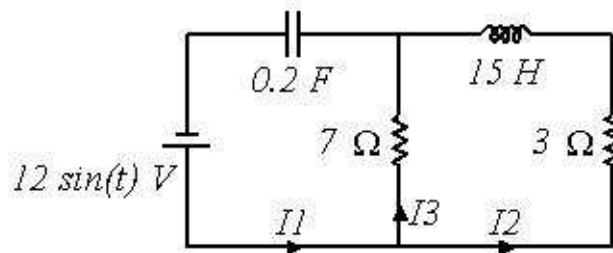Set up the **equations** for the system.
Do not solve the equations.
Give an **algebraic equation** for I_3 and
differential equations for I_1 and I_2.
The charge on the capacitor and I_3 should
NOT appear in the equations for I_1 and I_2.

4. (30 points)   Solve the system of differential equations and initial conditions.

$$\frac{dx}{dt} = -4x + 2y + 2 \qquad\qquad x(0) = \frac{7}{4}$$

$$\frac{dy}{dt} = 4x - 6y \qquad\qquad y(0) = \frac{9}{2}$$

HINTS: Write the equations in operator form.

Cross multiply by operators to separate the variables.

Use the characteristic polynomial to find the homogeneous solution for one variable.

Use undetermined coefficients to find the particular solution for that variable.

Compute the other variable.

Use the initial conditions to determine the constants of integration.

5. (30 points) Let $L(y) = y'' - (\tan x)y' - (\sec^2 x)y$. Find the general solution of the differential equation $L(y) = 3\sec^3 x \tan x$ by completing the following steps:

    a. Show $y_1 = \sec x$ and $y_2 = \tan x$ are solutions of the homogeneous equation $L(y) = 0$.

    b. Show $y_1 = \sec x$ and $y_2 = \tan x$ are linearly independent.

    c. Write out the general homogeneous solution.

    d. Use variation of parameters to find a particular solution.

    e. Write out the general non-homogeneous solution.

6. (15 points) Find the solution of the initial value problem:

$$\frac{d^3y}{dt^3} + 4\frac{dy}{dt} = 4\cos(2t) \quad \text{with} \quad y(0) = 2 \quad \text{and} \quad \frac{dy}{dt}(0) = \frac{1}{2} \quad \text{and} \quad \frac{d^2y}{dt^2}(0) = 4$$