Name____________________ ID__________

1	/20	3	/40
2	/40	4	/10

MATH 311 Exam 2 Fall 2000

Section 502 P. Yasskin

1. (20 points) Consider the vector space P_2 of polynomials of degree ≤ 2.
Consider the function of two polynomials

$$\langle p,q\rangle = \int_0^1 p(x)\,q(x)\,dx$$

a. (10 pts) Show $\langle p,q\rangle$ is an inner product.

b. (10 pts) Find $\cos\theta$ where θ is the angle between the polynomials

$$p = 5x^2 + 3 \quad \text{and} \quad q = 3x.$$

2. (40 points) Consider the vector space P_2 of polynomials of degree ≤ 2.
Consider the bases

$$e_1 = 1 \qquad e_2 = x \qquad e_3 = x^2$$

$$f_1 = 1 + x \qquad f_2 = x \qquad f_3 = -x + x^2$$

Consider the function $L : P_2 \to P_2$ given by

$$L(p) = 2p(0) + p(1)x$$

a. (5 pts) Show L is linear.

b. (5 pts) Find the matrix of L relative to the e-basis. Call it $\underset{e \leftarrow e}{A}$.

c. (10 pts) Find the change of basis matrices

- $\underset{e \leftarrow f}{C}$ from the f-basis to the e-basis, and

- $\underset{f \leftarrow e}{C}$ from the e-basis to the f-basis.

d. (10 pts) Find the matrix of L relative to the f-basis. Call it $\underset{f \leftarrow f}{B}$.

e. (5 pts) Find $\underset{f \leftarrow f}{B}$ by a second method.

f. (5 pts Extra Credit) What are the eigenvalues and corresponding eigenpolynomials of L?
(This required no computations.)

3. (40 points) Consider the matrix $A = \begin{pmatrix} -6 & -8 \\ 4 & 6 \end{pmatrix}$.

a. (15 pts) Find the eigenvalues and eigenvectors of A.

b. (10 pts) Find the diagonalizing matrix X so that $A = XDX^{-1}$ where D is diagonal. What are D and X^{-1}?

c. (5 pts) Find A^{10}.

d. (5 pts) Find e^{At}.

e. (5 pts Extra Credit) Find $\sin\left(\frac{\pi}{4}A\right)$.

HINT: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

4. (10 points) Let $V =$ Span$\{\vec{v}_1, \vec{v}_2\}$ where

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

Notice that V is a subspace of $\mathbf{R}^5$.

a. (6 pts) Find $V^{\perp}$ the orthogonal subspace to V.

b. (2 pts) What is a basis for $V^{\perp}$?

c. (2 pts) What is the dimension of $V^{\perp}$?