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1-5	/25	7	/11
6	/45	8	/24

MATH 311 Final Exam Fall 2000
Section 502 P. Yasskin

1. (5 points) Compute $\oint (4x-3y)\,dx+(3x-2y)\,dy$ counterclockwise around the edge of the triangle with vertices $(0,0)$, $(0,3)$ and $(2,0)$.
(HINT: Use Green's Theorem.)

a. 3
b. 6
c. 9
d. 12
e. 18

2. (5 points) Let $M(2,2)$ be the vector space of 2×2 matrices. Consider the linear map

$$L : M(2,2) \to M(2,2) \quad \text{given by} \quad L(X) = X + X^{\mathrm{T}}$$

where X^{T} is the transpose of X. Which of the following is FALSE or are they all TRUE?

a. Domain$(L) = M(2,2)$
b. Codomain$(L) = M(2,2)$
c. Kernel$(L) = \{$antisymmetric 2×2 matrices$\}$
d. Image$(L) = \{$symmetric 2×2 matrices$\}$
e. All of the above are TRUE

3. (5 points) If a jet flies around the world from East to West, directly above the equator, in what direction does the unit binormal $\hat{B}$ point?
(HINT: $\hat{B} = \hat{T}\times\hat{N}$)

a. North
b. South
c. East
d. Up (away from the center of the earth)
e. Down (toward the center of the earth)

4. (5 points) Let P_2 be the vector space of polynomials of degree at most 2. Consider the inner product

$$\langle p, q\rangle = \int_0^1 3x\, p(x)\, q(x)\, dx$$

Find the angle between the polynomials $r(x) = 1 - x^2$ and $s(x) = x$.

a. $\cos^{-1}\left(\frac{4\sqrt{2}}{5\sqrt{3}}\right)$

b. $\cos^{-1}\left(\frac{16}{15}\right)$

c. $\cos^{-1}\left(\frac{5\sqrt{3}}{4\sqrt{2}}\right)$

d. $\cos^{-1}\left(\frac{15}{16}\right)$

e. $\cos^{-1}\left(\frac{4\sqrt{3}}{5\sqrt{2}}\right)$

5. (5 points) Compute $\iint_{\partial C} \vec{F} \bullet d\vec{S}$ for $\vec{F} = (x^3, y^3, x^2 z + y^2 z)$ over the complete surface of the cylinder $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$.

(HINT: Use Gauss' Theorem.)

a. 24π

b. 48π

c. 96π

d. 144π

e. 324π

6. (45 points) Consider the vector space V spanned by

$$e_1 = \cosh^2 x, \quad e_2 = \sinh^2 x \quad \text{and} \quad e_3 = \cosh x \sinh x.$$

USEFUL FACTS:

$\sinh 0 = 0$	$\cosh^2 x - \sinh^2 x = 1$	$\frac{d}{dx}\sinh^2 x = 2\cosh x \sinh x$
$\cosh 0 = 1$		
$\frac{d}{dx}\sinh x = \cosh x$	$\sinh(2x) = 2\sinh x \cosh x$	$\frac{d}{dx}\cosh^2 x = 2\cosh x \sinh x$
$\frac{d}{dx}\cosh x = \sinh x$	$\cosh(2x) = \cosh^2 x + \sinh^2 x$	$\frac{d}{dx}\cosh x \sinh x = \cosh^2 x + \sinh^2 x$

a. (5 pts Extra Credit) Show e_1, e_2 and e_3 are linearly independent.

b. (10 pts) Another basisfor V is $f_1 = 1, \quad f_2 = \cosh(2x)$ and $f_3 = \sinh(2x)$. Find the change of basis matrices from the f-basis to the e-basis, $\underset{e \leftarrow f}{C}$ and from the e-basis to the f-basis, $\underset{f \leftarrow e}{C}$.

c. (5 pts) Consider the linear operator of differentiation

$$D : V \to V \quad \text{given by} \quad D(g) = \frac{dg}{dx}$$

Since

$$D(f_1) = D(1) = 0$$
$$D(f_2) = D(\cosh(2x)) = 2\sinh(2x) = 2e_3$$
$$D(f_3) = D(\sinh(2x)) = 2\cosh(2x) = 2e_2$$

the matrix of D relative to the f-basis is $\underset{f\leftarrow f}{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$.

Find the matrix of D relative to the e-basis. Call it $\underset{e\leftarrow e}{A}$.

d. (5 pts) Find the matrix of D relative to the e-basis by a second method.

e. (16 pts) Find the eigenvalues and eigenvectors of the matrix $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$.

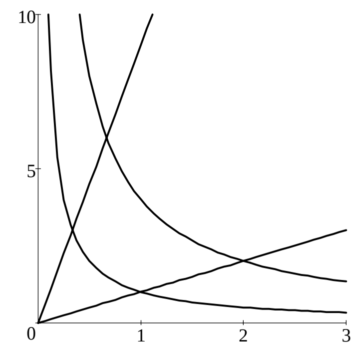
f. (4 pts) Find the eigenvalues and eigenfunctions of the operator D.

7. (11 points) Compute $\iint \frac{1}{x} e^{\sqrt{xy}}\, dx\, dy$ over the diamond shaped region between the curves

$$y = x, \qquad y = 9x, \qquad y = \frac{1}{x} \qquad \text{and} \qquad y = \frac{4}{x}.$$

You **must** use the curvilinear coordinates

$$x = \frac{v}{u} \qquad \text{and} \qquad y = uv.$$



a. (3 pts) Find the Jacobian:

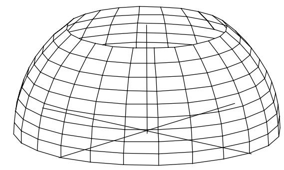
b. (4 pts) Express each boundary curve in terms of u and v:

c. (2 pts) Express the integrand in terms of u and v:

d. (2 pts) Compute the integral:

8. (24 points) Use two methods to compute

$$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$$



for $\vec{F} = (y, -x, z^2)$ over the piece of the sphere $x^2 + y^2 + z^2 = 25$ for $0 \le z \le 4$ with normal pointing away from the z-axis.

a. (12 pts) Parametrize the surface, compute $\vec{\nabla} \times \vec{F}$ and compute the double integral $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S}$ directly.

b. (12 pts) By Stokes' Theorem $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint_{\partial S} \vec{F} \bullet d\vec{s}$ where ∂S is the boundary of S.

Parametrize the upper and lower circles and compute $\oint \vec{F} \bullet d\vec{s}$ for each circle.

Be sure to discuss the orientation of the circles when you add up the integrals.