Name________________________ ID____________

Math 311 Exam 2 Spring 2002

Section 503 P. Yasskin

1	/10	5	/10
2	/15	6	/15
3	/10	7	/25
4	/15		

1. (10 points) Which one of the following is **NOT** a vector space? Why?

a. $Q = \{(w,x,y,z) \in \mathbf{R}^4 \mid w + 2x + 3y + 4z = 0\}$

b. $S = \{X \in M(2,2) \mid AXA = X\}$ where $M(2,2)$ is the set of 2×2 matrices and $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

c. $T = \{p \in P_3 \mid p(1) = p(0) + 1\}$ where P_3 is the set of polynomials of degree less than 3.

2. (15 points) For **ONE** of the two vector spaces listed in #1 (say which), give a basis and the dimension.

3. (10 points) Which one of the following is NOT a linear function? Why?

a. $I : C[0,2\pi] \to \mathbf{R}$ where $C[0,2\pi]$ is the set of continuous real valued functions on the interval $[0,2\pi]$ and $I(f) = \int_0^{2\pi} [x + f(x)]\,dx$.

b. $Z : M(2,2) \to M(2,2)$ given by $Z(X) = BXB$ where $B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$.

c. $E : P_3 \to P_3$ given by $E(p(x)) = x\dfrac{dp(x)}{dx} - p(x)$.

4. (15 points) For **ONE** of the two linear functions listed in #3 (say which), find the kernel and the image (as the span of some vectors) and determine if the function is onto and/or one-to-one.

5. (10 points) Let P_3 be the vector space of polynomials of degree less than 3. Which one of the following is NOT an inner product on P_3? Why? HINT: Let $p = a + bx + cx^2$ and find which is not positive definite.

a. $\langle p,q\rangle_1 = \int_0^1 xp(x)q(x)\,dx$

b. $\langle p,q\rangle_2 = p(0)q(0) + p(1)q(1)$

c. $\langle p,q\rangle_3 = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$

6. (15 points) For **ONE** of the two inner products listed in #5 (say which), find the angle between the polynomials

$$r(x) = 4x \qquad \text{and} \qquad s(x) = 6x^2$$

7. (25 points) Consider the vector space $V = Span\{\sin^2\theta, \cos^2\theta, \sin\theta\cos\theta\}$

a. (3) Show $e_1 = \sin^2\theta$, $e_2 = \cos^2\theta$, $e_3 = \sin\theta\cos\theta$ is a basis for V.

b. (7) Another basis is $f_1 = 1$, $f_2 = \sin 2\theta$, $f_3 = \cos 2\theta$. Find the change of basis matrices from the e-basis to the f-basis and vice versa. Be sure to say which is which.

(7 continued)

c. (4) Consider the derivative operator $D : V \to V$ given by $D(g) = \dfrac{dg}{d\theta}$.
Find the matrix of D relative to the e-basis.

d. (6) Find the matrix of D relative to the f-basis in **TWO** ways.

i. by using the matrix of D relative to the e-basis:

ii. by differentiating basis vectors:

(7 continued)

e. (5) Consider the function $g(\theta) = 5\sin 2\theta + 3\cos 2\theta$. Compute $D(g)$ in **TWO** ways:

i. by differentiating:

ii. by using the matrix of D relative to the *f*-basis: