Name________________________ ID____________

Math 311 Exam 3 Spring 2002

Section 503 P. Yasskin

Multiple Choice (8 points each.)

1-7	/56	9	/24
8	/20	10 EC	/10

Circle 3 to grade for part credit: 1 2 3 4 5 6 7

1. If $F = (xy, yz, xz)$ then $\vec{\nabla} \cdot \vec{F} =$

a. $y - z + x$
b. $(-y, z, -x)$
c. $x + y + z$
d. $(-y, -z, -x)$
e. $-x + y - z$

2. If $F = (xy, yz, xz)$ then $\vec{\nabla} \times \vec{F} =$

a. $y - z + x$
b. $(-y, z, -x)$
c. $x + y + z$
d. $(-y, -z, -x)$
e. $-x + y - z$

3. If $f(x,y,z) = x\sin(yz) - y\cos(xz) + z\tan(xy)$ then $\vec{\nabla} \times \vec{\nabla} f =$

a. $z\sin(yz)\vec{i} + z\cos(xz)\vec{j} + xy\sec^2(xy)\vec{k}$
b. $\sin(yz)\vec{i} - \cos(xz)\vec{j} + \tan(xy)\vec{k}$
c. $\cos(yz)\vec{i} + \sin(xz)\vec{j} + \sec^2(xy)\vec{k}$
d. 0
e. Does not exist.

4. Compute the line integral $\int y\,dx - x\,dy$ counterclockwise around the semicircle $x^2 + y^2 = 4$ from $(2,0)$ to $(-2,0)$. (HINT: Parametrize the curve.)

a. -4π
b. -2π
c. π
d. 2π
e. 4π

5. Compute the line integral $\int \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = \left(\frac{1}{x}, \frac{1}{y}\right)$ along the curve $\vec{r}(t) = \left(e^{\cos(t^2)}, e^{\sin(t^2)}\right)$ for $0 \le t \le \sqrt{\pi}$. (HINT: Find a potential f so that $F = \vec{\nabla} f$.)

a. -2
b. 0
c. $\frac{2}{e}$
d. 1
e. π

6. Compute $\oint (5x + 3y)\, dx + (x - 2y)\, dy$ counterclockwise around the edge of the rectangle $1 \le x \le 5$, $3 \le y \le 6$. (HINT: Use Green's Theorem.)

a. 36
b. 24
c. 12
d. –24
e. –36

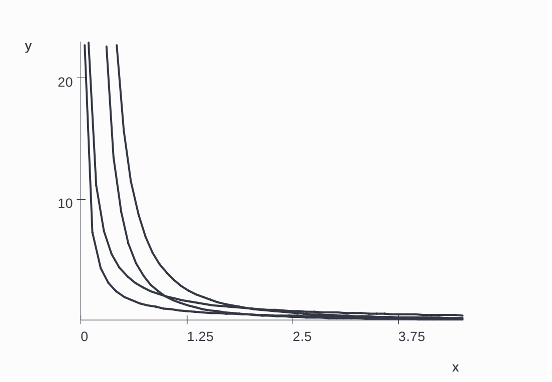
7. Compute $\iint_{\partial C} \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = \left(zx^3, zy^3, z^2(x^2 + y^2)\right)$ over the total surface of the solid cylinder $C = \left\{(x,y,z) \mid x^2 + y^2 \le 4,\ 0 \le z \le 3\right\}$ with outward normal. (HINT: Use Gauss' Theorem.)

a. 360π
b. 180π
c. 90π
d. 60π
e. 30π

8. (20 points) Compute $\iint_R x^2 y \, dx\, dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{2}{x}, \qquad y = \frac{2}{x^2}, \qquad y = \frac{4}{x^2}$$

For full credit you must use curvilinear coordinates. Half credit for rectangular coordinates.



9. (24 points) Stokes' Theorem states that if S is a nice surface in $\mathbf{R}^3$ and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S and $\vec{F}$ is a nice vector field on S then

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$

Verify Stokes' Theorem if $F = (-yx^2, xy^2, x^2 + y^2)$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ below $z = 2$ with normal pointing up and in.

9a. (4 points) Compute $\vec{\nabla} \times \vec{F}$. (HINT: Use rectangular coordinates.)

9b. (10 points) Compute $\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$.

(HINT: Here is the parametrization of the cone and the steps you should use. Remember to check the orientation of the surface.)

$\vec{R}(r, \theta) = (r\cos\theta, r\sin\theta, r)$

$\vec{R}_r =$

$\vec{R}_\theta =$

$\vec{N} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r, \theta)\right) =$

$\left(\vec{\nabla} \times \vec{F}\right) \cdot \vec{N} =$

$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

9c. (10 points) Compute $\oint_{\partial S} \vec{F} \cdot d\vec{s}$. Recall $F = (-yx^2, xy^2, x^2 + y^2)$.

(HINT: Parametrize of the boundary circle. Remember to check the orientation of the curve.)
(CHECK the answers to 9b and 9c agree.)

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\vec{F} \cdot \vec{v} =$

$\oint_{\partial S} \vec{F} \cdot d\vec{s} =$

10. (10 points Extra Credit) Compute the line integral $\oint \frac{y}{x^2+y^2}\,dx - \frac{x}{x^2+y^2}\,dy$ counterclockwise around the boundary of the plus sign shown below.

Be sure to justify any theorem you use. (Hint: The answer is not zero.)